A Political Economy Theory of Growth

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Abstract

The standard neoclassical growth model predicts that developing economies will eventually catch up with leading economies. While good performances from Asian countries support the standard neoclassical growth model, economic stagnation in Sub-Saharan Africa and Latin America calls for a different theory that is capable of explaining both growth miracles and growth tragedies. This paper shows that that a high degree of patience in the preferences of citizens and politicians and the ability of citizens to replace a politician in power are key ingredients for economic growth. I characterize the necessary conditions for growth to occur in a context where technological progress is available and free, but requires the approval of self-interested politicians to be adopted. In the model proposed, when politicians in power have a low discount factor, they find it optimal to stop technological progress in exchange for static rewards that the representative citizen does not control. The paper predicts that everything else equal, economies that are the most likely to grow are those with the strongest political institutions: the lowest probabilities of occurrence of a coup d'etat and the lowest probabilities of falling in an absorbing state of dictatorship. Consistently with empirical facts on growth, the relationship predicted between dictatorship and economic growth by the model is a non-linear one: given a probability of falling in the state of dictatorship, the occurrence of growth depends on the discount factor of citizens. The paper also shows that even when the economy is already growing as a dictatorship, a one-shot transition to democracy is still desirable to citizens as it reduces the payoffs that are necessary to provide dynamic incentives to politicians in power.

Introduction

The standard neoclassical growth model predicts that developing economies will eventually catch up with leading economies. While good performances from Asian countries support the standard neoclassical growth model, economic stagnation in Sub-Saharan Africa and Latin America calls for a different theory that is capable of explaining both growth miracles and growth tragedies. In a context where technological progress is available and free but requires action from politicians to be adopted by the economy, this paper shows that a high degree of patience in the preferences of citizens and politicians and the ability of citizens to replace a politician in power are key ingredients for economic growth.

This study proposes a model where nature shifts the technological frontier forward every period. However, the economy can take advantage of that shift only after it has been approved by a politician in office. A politician in power approves (or adopts) technological progress by processing an investment good. This in turn propagates growth in the economy. A politician in power who chooses not to process the investment good for economic growth may consume it and increase his/her own welfare. When technological progress is adopted, it benefits both citizens and politicians. However, politicians are self-interested and approve growth only when they are sufficiently compensated to make up for not consuming the investment good. Even when they are paid the maximum resources available, impatient politicians still find it beneficial to consume the investment good and let the economy stagnate. This paper studies subgame perfect equilibria of the game between citizens and politicians and focuses on the properties of best subgame perfect equilibria which are subgame perfect equilibria that maximize payoffs of citizens. This methodology is common in the principal-agent literature and is also used by Acemoglu, Golosov and Tsyvinski (2008) and Miquel & Yared (2010) who study the properties of an optimal contract between an agent who has an advantage over the accomplishment of a task and a principal who is willing to delegate this task. By focusing on best subgame perfect equilibria, this paper departs from suboptimalities that may simply be resolved through renegotiation.

Given this basic setup laid out in section 1, this paper further analyzes the impact of political instability and dictatorship on economic growth in sections 2 and 3 respectively. Political instability is modeled as an event"coup d'etat" whose occurrence is associated with the politician in power being overthrown and replaced with a different politician randomly chosen by nature. Dictatorship is defined as a state in which citizens are deprived of their voting rights and the incumbent ruler (or politician in power) is expected to remain in power forever. Such a ruler is referred to as a dictator. An economy ruled by a dictator is referred to as a dictatorial economy. The paper compares performances of a continuum of economies each characterized by a given probability of falling in the state of dictatorship. In the model proposed, democracies are referred to as economies where the probability of falling in the state of dictatorship is null.

The introduction of political instability scales the effective discount factor of politicians down by the probability that a coup d'etat does not occur. The logic is as follows. Because a coup d'etat ends the term of office of an incumbent politician, its likelihood makes expected-utility maximizing politicians short-sighted and willing to consume the investment good and let the economy stagnate. The paper shows that once political instability is introduced, an economy that would otherwise grow will now grow only if the likelihood of a coup d'etat is not too large.

The introduction of a probability to fall in the state of dictatorship has a similar effect on economic growth. The theory predicts that democratic economies that do not grow will also not grow for any given probability of falling in the state of dictatorship. On the other hand, dictatorial economies that are capable of growing will also grow for any given probability of falling in the state of dictatorship. For intermediate economies where agents are patient enough to allow for growth under democracy but not sufficiently patient to do so after the economy has become a dictatorship, the occurrence of growth depends on the probability of falling in the state of dictatorship. For this last class of economies, given identical preference parameters, economies that grow are those below a threshold probability of falling in the state of dictatorship. The leading mechanism behind this result is the following. In the best subgame perfect equilibria, rulers of democratic economies fear replacement while dictators do not, as they are guaranteed an eternal term of office. Dictators therefore have a higher bargaining power compared to rulers of non-dictatorial economies. In fact, the higher the probability of falling in the state of dictatorship, the higher the bargaining power of a ruler. It follows that everything else constant, compensations that make politicians willing to approve growth increase with the probability of falling in the state of dictatorship. In other words, given identical preference parameters, a dictatorial economy is more prone to stagnation than a democratic economy. At the same time, remunerations to politicians tend to increase with the probability of falling in the state of dictatorship. That is, citizens are better off as the economy shifts toward democracy since in this case, growth becomes more likely while payoffs to citizens naturally increase. Yet, as emphasized earlier, a dictatorial economy may perform better than a democratic economy if its agents are more patient than those of the latter. In short, the relationship between democracy and economic growth is a non-linear one.

An important limitation of the theory proposed in this paper is that it does not lend itself fully to empirical tests, given the lack of reliable data to estimate discount factors in poor countries. However, the theory fits the empirical evidence relatively well in several other dimensions.

First, using a worldwide data set on national leaders from 1945 to 2000, and restricting to years of transition where the end of the leader's rule was caused by death due to a natural cause or to an accident, Jones & Olken (2005) conclude that a one standard deviation change in leader quality leads to a growth change of 1.5 percentage points per year. Their finding suggests that the nature of a leader and perhaps his/her time preferences are key determinants of economic growth.

Second, the theory proposed in this paper predicts that impatient politicians ask for large compensations in order to approve growth. One should therefore expect that everything else equal, countries with the largest government consumption expenditures per capita are also those with the most impatient politicians and the smallest growth rates. This prediction is confirmed by the finding of Barro (1991) from cross-country regressions that the ratio of government consumption expenditure to GDP is inversely related to per-capita growth.

Third, this paper suggests that the frequency of coups d'etat is a negative determinant of growth. In a sample of 113 countries studied between 1950 and 1982, Alesina, Ozler, Roubini and Swagel (1996) find that the average frequencies of coups d'etat for Latin America, Africa, Asia and industrial countries were 0.079, 0.060, 0.037 and 0 respectively, while the sample average was 0.048. Over the period, the average annual growth rates for Latin America, Africa, Asia and industrial countries were 2.2%, 1.4%, 3.3% and 2.9% respectively. Using a structural equation system to control for simultaneity and reverse causality, they find strong evidence that a high frequency of coups d'etat deters economic growth. This result of strong causality from political instability to economic growth is also reported by Barro (1991) and Easterly & Rebelo(1993). Alesina, Ozler, Roubini and Swagel (1996) explain this result by arguing that a high level of political instability implies uncertain future policies which in turn encourage risk-averse economic agents to wait to take productive initiatives or to exit the economy by investing abroad. The current paper suggests a new channel through which political instability affects economy growth: by making it difficult to provide correct incentives to opportunistic politicians who become short-sighted as the economy becomes politically unstable.

Fourth, this paper predicts that democratization fosters economic growth. This result is validated by Persson and Tabellini (2006) who from cross-country regressions find that becoming a democracy accelerates growth by 0.75 percentage points.

Finally, the theory proposed in this paper predicts that the relationship between dictatorship and economic growth is a non-linear one. A dictatorship may grow or stagnate depending on how patient the dictator is. This implication is consistent with the empirical findings of Alesina, Ozler, Roubini and Swagel (1996) who conduct cross-country regressions and fail to identify a linear relationship between democracy and growth. Alesina, Ozler, Roubini and Swagel (1996) define democracy as a variable that takes on value 1 for countries with " free competitive general elections with more than one party running " , on value 2 for countries " with some forms of elections but with very severe limits in the competitiveness of such ballots " , and on value 3 for " countries in which leaders are not elected " . They find that over the period studied, the average value of democracy was 2.18, 2.82, 2, 32 and 1.07 for Latin America, Africa, Asia and industrial countries respectively and conclude that there is no obvious relationship between democracy and growth. Alesina, Ozler, Roubini and Swagel (1996) explain this finding by two observations. First, lobbyism encourages policy makers in democratic regimes to favor opportunistic policies that are detrimental to growth, while dictators free from competition may be less sensitive to it. However, because in some instances dictators may also need to be opportunistic if their survival is not secured, it is not clear which of democracy and dictatorship is more favorable to growth. Second, as emphasized by the authors:

> *"authoritarian regimes are not a homogenous lot: they include technocratic dictators and kleptocratic ones. While the apparent association of high economic growth with authoritarian regimes is suggested by the experience of several authoritarian technocratic regimes (such as those in Korea, Taiwan,Indonesia, Turkey, Chile and so on), it is as well evident that for each benevolent dictator, one can*

observe at least as many kleptocratic or inept authoritarian regimes whose rule led to systematic economic mismanagement and eventual political and economic collapse of their countries".

This paper goes a step further to characterize kleptocratic and technocratic dictators while highlighting important mechanisms through which the nature of political institutions is linked to economic growth.

Brief Review of The Literature

There exists a large body of literature which seeks to understand divergences in economic performances across countries. Parente and Prescott (1999) explain these divergences by the existence of barriers such as unions which in some countries protect inefficient work practices at the firm level. Krusell and Rios-Rull (1996) propose a vintage human capital model where agents either accumulate skills related to existing or new technologies and become managers, or work as unskilled for their entire lifetimes. In their model, innovation shifts demand away from managers of current technologies toward managers of new and cheaper technologies, while raising the purchasing power of the unskilled. It results that the unskilled are in favor of innovation while the skilled are against it. In that model, every period, each agent votes for either laissez-faire (which allows the development of new technologies), or the prohibition of new technologies and the majority wins. The authors find that depending on the initial distribution of skills, the economy may converge to a long-term equilibrium with permanent growth or to a long-term equilibrium with economic stagnation.

Benhabib & Rustichini (1996) suggest a theory which predicts a positive relationship between the initial stock of physical capital of a country and its growth rate when the utility function of agents is sufficiently concave. In the model of Benhabib and Rustichini (1996), organized social groups independently choose consumption levels which are turned into actual consumptions according to a preestablished allocation rule. Residual output is then accumulated as capital for the following period. Benhabib Rustichini (1996) characterize subgame perfect equilibrium outcomes of this game between the social groups using trigger strategies which threaten to shift to an undesirable outcome after any deviation from a first best outcome featuring sustained growth. Their study finds that at high initial capital stocks where consumption levels are high and marginal utilities low, the social groups are not willing to deviate from the first best arrangement. However, at low initial capital stocks associated with low consumption levels and high marginal utilities, the social groups are more willing to deviate at the expense of future retaliation, especially when marginal productivity of capital is not very high at low wealth levels. The theory of Benhabib and Rustichini (1996) is consistent with the empirical finding of Fisher (1991) that investment rates in physical capital are positively correlated with income levels.

Agarwala (1983) finds that distortions in market prices explain a significant proportion of differences in economic performances among countries. Based on corruption indexes provided by non-governmental organizations that monitor countries, Mauro (1995) reports that corruption is a negative determinant of economic growth. Easterly & Levine (1997) argue that ethnic diversity creates polarization and encourages opportunistic behaviors and find that it accounts for more than 28 % of the growth differential between the countries of Africa and East Asia. Alesina, Ozler, Roubini and Swagel (1996) find evidence that political instability negatively impacts economic growth. Barro (1991) concludes from cross-country regressions that among poor countries with the same initial GDP per capita, those that catch up the fastest are those with the highest initial levels of human capital per capita. He finds that the initial level of human capital, per capita government consumption expenditures, political instability (proxied by figures on revolutions, coups and political assassinations), price distortions (based on purchasing-power parity numbers for investment deflators) and the nature of the economic system (market versus not market-oriented) are important determinants of economic growth. Yet, the author reports that these factors together do not fully explain the relatively weak growth performances of countries in Sub-Saharan Africa and Latin America.

Despite existing evidence that political instability negatively affects economic growth, rigorous theoretical analyses on the subject remain scarce. Cukierman, Edwards, & Tabellini (1992) study the impact of political instability and polarization on seigniorage in a model economy with two political parties. They define political turnover to be governed by a Markov process whose transition probability measures political instability. They describe polarization among political parties as disagreement over the composition of a public good that the parties value. In their model, the political party in office in entrusted with choices of the current period level of seigniorage and tax rate (fiscal policy), as well as the next period efficiency level of the tax system (tax reform). They find that when the probability of political turnover or the degree of polarization is very high, it is optimal for the political party in office to choose an inefficient tax system in order to discourage future governments from collecting taxes and spending them on the goods that the party in office does not value. The authors succeed in validating these predictions with econometric regressions and conclude that highly polarized and politically unstable countries rely more heavily on seigniorage to finance public good consumption. Given that high inflation is negatively associated with economic growth (De Gregorio (1992)), the work of Cukierman, Edwards, & Tabellini (1992) may be understood as a theory of economic growth and political instability. Svensson(1998) uses a model similar to that of Cukierman, Edwards, & Tabellini (1992) to study the impact of polarization and political instability on the quality of the legal environment. In that model, a strong legal environment is associated with a high level of enforcement of property rights and a high level of private investment. Arguing that a high turnover rate prevents an incumbent government from fully internalizing the benefits of a legal reform, Svensson(1998) concludes that high political instability negatively affects private investment. Using cross-country regressions, Svensson(1998) finds that political instability has a negative and significant impact on private investment when quality of property rights is not accounted for. He also finds that quality of property rights has a positive and significant effect on private investment when political instability is not controlled for. However, when both political instability and quality of property rights are controlled for, the author finds that only the effect of quality of property rights remains significant. From these results, Svensson(1998) concludes that political instability affects private investment essentially by discouraging governments from reforming the legal system to improve the quality of property rights. In both papers however, the political process is exogenous and not sophisticated enough to capture the key mechanisms delivered by the current paper.

The methodology used in this paper builds on the work of Acemoglu, Golosov & Tsyvinski (2008) who study a principal-agent political economy model where citizens hire and pay politicians to implement the best outcome of the economy. They study a model with double-sided commitment and find that if politicians are more patient than citizens, then political-economy induced distortions will disappear in the long-term. The political-economy setup of this paper is essentially theirs. However, this paper differs fundamentally from theirs in three respects. First, the current study analyzes technology adoption and links it to growth, while their paper focuses on capital accumulation and does not relate it to growth. Second, this paper models political instability and dictatorship which are absent in theirs. Third, their paper assumes the existence of a commitment technology from which this paper abstracts. In the context of underdeveloped economies, the assumption of a commitment technology would be difficult to justify as it would deny the true nature of these economies which typically function under weak institutions.

1 The Basic Setup

This section studies the impact of patience in the preferences of economic agents on growth. The model economy is populated with identical citizens and identical politicians. Preferences and technology are first described. The game is then studied and conditions for growth are laid out in the section's main theorem.

1.1 The Model

The model economy is populated with identical citizens and an infinite number of identical politicians who belong to a set J. A representative citizen is assumed to make decisions on behalf of all other citizens. Each period, the representative citizen is endowed with 1/2 unit of a non-storable consumption good that can be readily consumed and 1/2 unit of a non-storable investment good that cannot be consumed before being processed. There exists two technologies for processing the investment good: it can be either transformed on a one-to-one basis into a non-storable consumption good that only politicians can consume, or it can be used to scale tomorrow's technological frontier forward for all economic agents. Politicians do not receive endowments. Only a politician in power possesses the technology for processing the investment good in either way. If no politician is hired to process the investment good, then the period 1/2 unit of endowment of investment good will be lost and the technology will stagnate.

1.2 Preferences

All politicians have common discount factor δ and all citizens have common discount factor β. S_t denotes the date t technological frontier. C_t^{HH} and C_t^{P} denote period t total consumptions for the representative citizen and an arbitrary politician. Later on, c_t^{HH} and c_t^P will be used to denote specific consumptions of the consumption good. For a politician in power, total consumption is the sum of consumptions of the investment good and the consumption good. Preferences are given by $\sum_{t=0}^{\infty} \beta^t \lambda^{S_t} U(C_t^{HH})$ for the representative citizen and $\sum_{t=0}^{\infty} \delta^t \lambda^{S_t} V(C_t^P)$ for a politician.

The following assumptions are made about preferences[1]:

A.1 U and V are continuous, strictly increasing, strictly concave and differentiable functions defined on $\mathbb{R}$

A.2 $\lambda > 1$

A.3 $\beta, \delta \in [0, \frac{1}{\lambda})$

A.3 $U(0) = V(0) = 0$ and $\lim\limits_{c \to +\infty} V(c) = +\infty$.

1.3 Technology

θ_t is a variable which takes on value 1 when either the representative citizen chooses not to hire any politician at date t or the politician in power chooses to process the investment good into consumption for himself, and which takes on value 0 when the politician in power chooses to process the investment good for growth. Given S_0 given, the law of motion of the technological frontier is given by:

$$S_{t+1} = S_t + 1 - \theta_t.$$

Under functional forms $U(c) = \frac{c^{1-\epsilon_1}}{1-\epsilon_1}$ and $V(c) = \frac{c^{1-\epsilon_2}}{1-\epsilon_2}$, it is straightforward that $U(\hat{\lambda}^{S_t} c) = (\hat{\lambda}^{1-\epsilon_1})^{S_t} U(c)$ and $V(\hat{\lambda}^{S_t} c) = (\hat{\lambda}^{1-\epsilon_2})^{S_t} V(c)$. It follows that with constant relative risk aversion (CRRA) functional forms for U and V, given constant streams c^{HH} and c^P, the model period utility functions are $\lambda^{S_t} U(c^{HH}) \equiv U((\lambda^{\frac{1}{1-\epsilon_1}})^{S_t} c^{HH})$ and $\lambda^{S_t} V(c^P) \equiv V((\lambda^{\frac{1}{1-\epsilon_2}})^{S_t} c^P)$. That is, in a stationary model economy with constant streams c^{HH} and c^P and with CRRA functional forms, $(\lambda^{\frac{1}{1-\epsilon_1}})^{1-\theta_t}$ and $(\lambda^{\frac{1}{1-\epsilon_2}})^{1-\theta_t}$ may be interpreted as the gross growth rates of consumption between dates t and $t+1$ for the citizens and the politicians respectively. If U and V are not CRRA utility functions, then for constant streams c^{HH} and c^P, $\lambda^{S_t} U(c^{HH})$ and $\lambda^{S_t} U(c^{HH})$ should be interpreted to reflect a taste for quality as in the quality-ladder model of Grossman and Helpman (1991). In this latter case, between any dates t and $t+1$, quality jumps by factor $\lambda^{1-\theta_t}$ which defines the gross growth rate of the economy.

[1] Assumption **A.3** is not crucial for future results. It is made for convenience only. All the results of this paper continue to hold when U and V are restricted to be defined on the set of positive real numbers only.

1.4 Timing of the Stage Game

The economy starts date 0 with politician $\iota_0 \in J$ in power and with technological frontier S_0. Then, after receiving endowments of the consumption and investment goods, the representative citizen decides which share of the consumption good to give to politician ι_0. Specifically, the representative citizen chooses an allocation $\{c_0^{HH}, c_0^P\}$ of the endowment of the consumption good s.t: $c_0^{HH} + c_0^P = 1/2$. Thereafter, politician ι_0 moves and chooses the value of θ_0, by deciding whether to consume the investment good or to process it for economic growth. Consumption occurs afterward, the representative citizen consuming c_0^{HH} and arbitrary politician $\iota \in J$ consuming $c_0^P + \frac{\theta_0}{2}$ if in power and 0 if not in power. That is, total consumptions at date 0 are $C_0^{HH} = c_0^{HH}$ for the representative citizen and

$$C_0^P = \begin{cases} c_0^P + \frac{\theta_0}{2} & \text{for a politician in power} \\ 0 & \text{for a politician not in power.} \end{cases}$$

The representative citizen then moves once again, this time choosing whether or not to replace politician ι_0. Specifically, the representative citizen chooses ι_1 in $J \cup \{\varnothing\}$ with ι_1 potentially equal to ι_0 or $\varnothing$. $\iota_1 = \varnothing$ implies that there will be no politician in power at date 1. In this case, θ_1 is necessarily equal to 1. The economy evolves in a similar way in period 1 and in all future periods. Any date t starts with states $\iota_t \in J \cup \{\varnothing\}$ and S_t that denote the politician in power and the technological frontier respectively.

1.5 Relevant Histories, Strategies and Equilibrium Concept

Let $h_{c,1}^t$, h_P^t, $h_{c,2}^t$, denote the date t histories available to the representative citizen in the first stage, to the politician in power in the second stage and to the representative citizen in the third stage of the period t game. Histories evolve as follows: $h_{c,1}^0 = \{\iota_0, S_0\}$, $h_P^0 = \{h_{c,1}^0, c_0^{HH}, c_0^P\}$, $h_{c,2}^0 = \{h_P^0, \theta_0, S_1\}$, $h_{c,1}^1 = \{h_{c,2}^0, \iota_1\}$ and so on...

Let $H_{c,1}^t|h^t$, $H_P^t|h^t$, $H_{c,2}^t|h^t$, denote the sets of all possible histories that the representative citizen in the first stage, the politician in power in the second stage and the representative citizen in the third stage may reach starting from some history h^t with states $\{S_t, \iota_t\}$. Also, define:

$$\Lambda \equiv \{(c_t^H, c_t^P) \in [0, 1/2]^2 \ \ s.t. \ \ c_t^H + c_t^P = 1\}. \tag{1}$$

Then, in the continuation game that follows h^t, players'strategies for the representative citizen in the first stage, the politician in power in the second stage and the representative citizen in the third stage are the following measurable functions:

$$\sigma_{c,1}|h^t : H_{c,1}^t|h^t \mapsto \Lambda \tag{2a}$$

$$\sigma_P|h^t : H_P^t|h^t \mapsto \{0; 1\} \tag{2b}$$

$$\sigma_{c,2}|h^t : H_{c,2}^t|h^t \mapsto J \cup \{\varnothing\} \tag{2c}$$

Let $\Sigma_{c,1}|h^t$, $\Sigma_{c,2}|h^t$, $\Sigma_P|h^t$ denote the sets of all continuation strategies after history h^t for the respective players. Given history h^t with states $\{S_t, \iota_t\}$, continuation strategies $\sigma_{c,1}|h^t$, $\sigma_{c,2}|h^t$ and $\sigma_P|h^t$ induce a sequences of allocations in the natural way. Let $\sigma|h^t \equiv (\sigma_{c,1}|h^t, \sigma_P|h^t, \sigma_{c,2}|h^t)$ be a continuation strategy profile that follows history h^t with states $\{S_t, \iota_t\}$ and that induces the sequence of allocations $\{c^{HH}_{t+\tau}, c^P_{t+\tau}, \theta_{t+\tau}, \iota_{t+\tau+1}\}_{\tau=0}^{\infty}$. Resulting payoffs from $\sigma|h^t$ to the citizens and to politician ι_t are respectively given by:

$$\Phi_c(\sigma|h^t, S_t, \iota_t) = \sum_{\tau=0}^{\infty} \beta^\tau \lambda^{S_{t+\tau}} U[c^{HH}_{t+\tau}] \tag{3a}$$

$$\Phi_{\iota_t}(\sigma|h^t_P, S_t, \iota_t) = \sum_{\tau=0}^{\infty} \delta^\tau \lambda^{S_{t+\tau}} V[c^P_{t+\tau} + \frac{\theta_{t+\tau}}{2}] * \mathbf{1}_{\{\iota_{t+\tau}=\iota_t\}}. \tag{3b}$$

Definition 1 *A Strategy profile $\sigma|\{S_0, \iota_0\}$ is a subgame perfect equilibrium (SPE) of the game if $\forall h^t$ with states $\{S_t, \iota_t\}$, the induced continuation strategies satisfy:*

$$\Phi_c(\sigma|h^t) \geq \Phi_c(\gamma, \sigma_P|h^t, \sigma_{c,2}|h^t), \forall \gamma \in \Sigma_{c,1}|h^t \tag{4a}$$

$$\Phi_{\iota_t}(\sigma|h^t) \geq \Phi_P(\sigma_{c,1}|h^t, \gamma, \sigma_{c,2}|h^t), \forall \gamma \in \Sigma_P|h^t \tag{4b}$$

$$\Phi_c(\sigma|h^t) \geq \Phi_{c,2}(\sigma_{c,1}|h^t, \sigma_P|h^t, \gamma), \forall \gamma \in \Sigma_{c,2}|h^t. \tag{4c}$$

Definition 2 *A sequence of allocations $\{c^{HH}_\tau, c^P_\tau, \theta_\tau, \iota_\tau\}_{\tau=0}^{\infty}$ is feasible if it satisfies: $c^{HH}_\tau + c^P_\tau = 1/2$, $\theta_\tau \in \{0; 1\}$ and $\iota_\tau \in J \cup \{\varnothing\}, \forall \tau \geq 0$.*

Lemma 3 *A feasible sequence of allocations $\{c^{HH}_\tau, c^P_\tau, \theta_\tau, \iota_\tau\}_{\tau=0}^{\infty}$ is an SPE sequence iff*

$$\sum_{\tau=0}^{\infty} \beta^\tau \lambda^{S_{t+\tau}} U[c^{HH}_{t+\tau}] \geq \frac{\lambda^{S_t}}{1-\beta} U(1/2), \forall t \geq 0 \tag{5a}$$

$$\sum_{\tau=0}^{\infty} \delta^\tau \lambda^{S_{t+\tau}} V[c^P_{t+\tau} + \frac{\theta_{t+\tau}}{2}] * 1_{\iota_{t+\tau}=\iota_t} \geq \lambda^{S_t} V[c^P_t + \frac{1}{2}], \forall t \geq 0, \tag{5b}$$

S_0 given and $S_{t+1} = S_t + 1 - \theta_t$.

Proof. ($\Rightarrow$) Suppose $\{c^{HH}_\tau, c^P_\tau, \theta_\tau, \iota_\tau\}_{\tau=0}^{\infty}$ is the sequence of allocations induced by some arbitrary strategy profile σ. The goal is to show that conditions 5(a) and 5(b) above hold. Let $\gamma_{c,1}|h^t$ be the strategy of the representative citizen that calls for choosing $\hat{c}^{HH}_{t+\tau} = \frac{1}{2}$ $\forall$ $\tau \geq 0$. Then, $\Phi_c(\sigma|h^t) \geq \Phi_c(\gamma_{c,1}|h^t, \sigma_P|h^t, \sigma_{c,2}|h^t) \geq \lambda^{S_t} U(\frac{1}{2}) + \frac{\beta}{1-\beta} \lambda^{S_t} U(\frac{1}{2})$, where the first inequality comes from the fact that $\sigma|h^t$ is a continuation strategy and the last inequality comes from the definition of the deviation strategy. This shows that condition 5(a) holds.

Now, define $\gamma_P|h^t$ as the strategy of a politician in power at date t that calls for choosing $\hat{\theta}_t = 1$ today and $\hat{\theta}_{t+\tau} = 1, \forall\, \tau > 0$, conditional on being in power at date $t+\tau$. By a similar argument to the one above, $\Phi_P(\sigma|h^t) \geq \Phi_P(\sigma_{c,1}|h^t, \gamma_P|h^t, \sigma_{c,2}|h^t) \geq \lambda^{S_t} V[(c_t^{HH} + \frac{1}{2})]$, so that condition 5(b) holds as well.

($\Leftarrow$) Now, suppose 5(a) and 5(b) hold for given sequence $\{c_\tau^{HH}, c_\tau^P, \theta_\tau, \iota_\tau\}_{\tau=0}^{\infty}$. It needs to be shown that there exists some SPE strategy profile that induces the sequence of allocations $\{c_\tau^{HH}, c_\tau^P, \theta_\tau, \iota_\tau\}_{\tau=0}^{\infty}$. For this purpose, let's define a trigger strategy profile σ' with the following properties. If all players have so far followed script $\{c_\tau^{HH}, c_\tau^P, \theta_\tau, \iota_\tau\}_{\tau=0}^{\infty}$, then σ' calls for continuing to follow the script. If any player has ever deviated in the past, then the representative citizen and all politicians are called to move to the worst outcome of the game forever: for all future periods, the representative citizen never hires any politician in power and consumes the total endowment of the consumption good; any politician ever hired always chooses to consume the investment good and let the technology stagnate. Clearly, σ' induces $\{c_\tau^{HH}, c_\tau^P, \theta_\tau, \iota_\tau\}_{\tau=0}^{\infty}$ on path. It is therefore left to show that σ' is an SPE strategy profile.

Let h^t denote a date t history with states S_t and ι_t, for some $t \geq 0$. Note that by 5(a), $\Phi_c(\sigma'|h^t) \equiv \sum_{\tau=0}^{\infty} \beta^\tau \lambda^{S_{t+\tau}} U[c_{t+\tau}^{HH}] \geq \frac{\lambda^{S_t}}{1-\beta} U(1/2)$. But $\frac{\lambda^{S_t}}{1-\beta} U(1/2)$ corresponds to the payoff to the first stage representative citizen when he/she chooses his/her best deviation ($c_t^{HH} = \frac{1}{2}$) at date t while all other players continue to follow the trigger strategy profile σ'. Hence, in the first stage of the game, the representative citizen never finds it profitable to deviate unilaterally from σ'. Similarly, by 5(b), $\Phi_P(\sigma'|h^t) = \sum_{\tau=0}^{\infty} \delta^\tau \lambda^{S_{t+\tau}} V[c_{t+\tau}^P + \frac{\theta_{t+\tau}}{2}] * 1_{\iota_{t+\tau}=\iota_t} \geq \lambda^{S_t} V[c_t^P + \frac{1}{2}]$, where the last term represents the payoff to the politician in power when he/she chooses his/her best deviation at date t ($\theta_t = 1$) while the representative citizen continues to follow the trigger strategy profile σ'. It follows that the politician in power never finds it profitable to deviate unilaterally from σ'.

To end the proof, it is left to check that the representative citizen does not want to unilaterally deviate from $\sigma'|h^t$ in the third stage of the period game. The payoff to the representative citizen in the third stage of the game if he/she chooses politician $\hat{\iota}_t$ in power instead of politician ι_t as indicated by script $\{c_\tau^{HH}, c_\tau^P, \theta_\tau, \iota_\tau\}_{\tau=0}^{\infty}$ is $\lambda^{S_t} U(c_t^{HH}) + \frac{\beta}{1-\beta} \lambda^{S_t} U(\frac{1}{2})$. But, by 5(a), $\Phi_c(\sigma'|h^t) = \sum_{\tau=0}^{\infty} \beta^\tau \lambda^{S_{t+\tau}} U[c_{t+\tau}^{HH}] \geq \frac{\lambda^{S_t}}{1-\beta} U(1/2) \geq \lambda^{S_t} U(c_t^{HH}) + \frac{\beta}{1-\beta} \lambda^{S_t} U(\frac{1}{2})$. This implies that the third stage-representative citizen does not want to unilaterally deviate from $\sigma'|h^t$ at date t, concluding the proof that $\sigma'|h^t$ is an SPE strategy profile. This ends the proof of lemma 3.

■

Lemma 4 *If an SPE sequence involves a replacement of the initial politician, then there exists another SPE sequence with no replacement that yields the same payoffs to citizens.*

Proof.

Let $\{c_\tau^{HH}, c_\tau^P, \theta_\tau, \iota_\tau\}_{\tau=0}^{\infty}$ be an SPE sequence that calls for one or several replacements of politicians in power. Now, consider the new sequence of allocations $\{c_\tau^{HH}, c_\tau^P, \theta_\tau, \iota_0\}_{\tau=0}^{\infty}$ which differs from the initial sequence only in that it maintains initial politician ι_0 in power forever. Clearly, at any date, this new sequence yields the same payoff to the representative citizen as the initial sequence. It is therefore left to show that the new sequence is also an SPE sequence. Clearly, because $\{c_\tau^{HH}, c_\tau^P, \theta_\tau, \iota_\tau\}_{\tau=0}^{\infty}$ is an SPE sequence, the previous lemma implies that $\sum_{\tau=0}^{\infty} \beta^\tau \lambda^{S_{t+\tau}} U[c_{t+\tau}^{HH}] \geq \frac{\lambda^{S_t}}{1-\beta} U(1/2), \forall t \geq 0$. The previous lemma also implies that $\sum_{\tau=0}^{\infty} \delta^\tau \lambda^{S_{t+\tau}} V[c_{t+\tau}^P + \frac{\theta_{t+\tau}}{2}] * 1_{\iota_{t+\tau}=\iota_t} \geq \lambda^{S_t} V[c_t^P + \frac{1}{2}], \forall t \geq 0$, which in turn implies $\sum_{\tau=0}^{\infty} \delta^\tau \lambda^{S_{t+\tau}} V[c_{t+\tau}^P + \frac{\theta_{t+\tau}}{2}] \geq \lambda^{S_t} V[c_t^P + \frac{1}{2}], \forall t \geq 0$. It therefore follows from lemma 4 that the new sequence of allocations is also an SPE sequence. This ends the proof of the lemma. ■

In what follows, best SPE sequences are referred to as SPE sequences that among all SPE sequences , yield the highest payoff to citizens. Lemma 4 implies that to characterize payoffs induced by such sequences, it is legitimate to restrict attention to SPE sequences that involve no replacement of the initial politician. Therefore, by lemma 3, the best SPE problem of this economy may be written as:

$$\max_{\{\{c_\tau^{HH}, c_\tau^P\} \in \Lambda, \theta_\tau \in \{0;1\}\}_{\tau=0}^{\infty}} \sum_{\tau=0}^{\infty} \beta^\tau \lambda^{S_\tau} U[c_\tau^{HH}]$$

s.t

$$\sum_{\tau=0}^{\infty} \beta^\tau \lambda^{S_{t+\tau}} U[c_{t+\tau}^{HH}] \geq \frac{\lambda^{S_t}}{1-\beta} U(1/2), \forall t \geq 0 \tag{6a}$$

$$\sum_{\tau=0}^{\infty} \delta^\tau \lambda^{S_{t+\tau}} V[c_{t+\tau}^P + \frac{\theta_{t+\tau}}{2}] \geq \lambda^{S_t} V[c_t^P + \frac{1}{2}], \forall t \geq 0 \tag{6b}$$

$$S_0 \text{ given }, S_{t+1} = S_t + 1 - \theta_t. \tag{6c}$$

Lemma 5 *There exists a best SPE sequence that is stationary:* $\{c_\tau^{HH}, c_\tau^P, \theta_\tau\} = \{c_\tau^{*HH}, c_\tau^{*P}, \theta_\tau^*\}$, $\forall \tau \geq 0$.

Proof.

The proof immediately follows from observing that the best SPE problem has a stationary structure in the sense that in any solution, citizens receive the same discounted sum of future utilities, in all periods. Details are given in the appendix . ■

Define $\phi^* = \max_{c_P \in \mathbb{R},\theta \in \{0;1\}} \frac{U(\frac{1}{2}-c_P)}{1-\beta\lambda^{1-\theta}}$ s.t $\frac{V[c_P+\frac{\theta}{2}]}{1-\delta\lambda^{1-\theta}} \geq V[c_P+\frac{1}{2}]$
and let argmax ϕ denote $\{c^*_{HH}, c^*_P, \theta^*\}$ such that $\phi^* = \frac{U(\frac{1}{2}-c^*_P)}{1-\beta\lambda^{1-\theta^*}}$ and $c^*_{HH} = \frac{1}{2} - c^*_P$.
Then, the stationary best SPE sequence $\{c^*_{HH}, c^*_P, \theta^*\}$ is characterized by:

$$\{c^*_{HH}, c^*_P, \theta^*\} = \begin{cases} \text{argmax}\,\phi & \text{if } \Phi^* \geq \frac{U(1/2)}{1-\beta} \\ \{1/2, 0, 1\} & \text{if } \Phi^* < \frac{U(1/2)}{1-\beta}. \end{cases}$$

Theorem 6 *There exists $\underline{\delta} < \frac{1}{\lambda}$ such that for $\delta \leq \underline{\delta}$, the economy does not grow. For $\delta > \underline{\delta}$, there exists $\beta^*(\delta)$ such that the economy grows if and only if $\beta > \beta^*(\delta)$. Moreover, $\beta^*(\delta)$ is a decreasing function of δ.*

Proof.

Define function h by

$$h(c_P) = \frac{V(c_P)}{V(c_P+\frac{1}{2})}. \tag{7}$$

Since V is continuous, so is h. In fact, h is strictly increasing and therefore invertible as a continuous bijection. To see why, observe that the derivative of h is given by:

$$h^{'}(c_P) = \frac{V^{'}(c_P)V(c_P+\frac{1}{2}) - V^{'}(c_P+\frac{1}{2})V(c_P)}{V(c_P+\frac{1}{2})^2}.$$

But by strict concavity of V, $V^{'}(c_P+\frac{1}{2}) \leq V^{'}(c_P)$. Therefore, $V^{'}(c_P)V(c_P+\frac{1}{2}) - V^{'}(c_P+\frac{1}{2})V(c_P) \geq V^{'}(c_P+\frac{1}{2})V(c_P+\frac{1}{2}) - V^{'}(c_P+\frac{1}{2})V(c_P) > 0$ follows from the fact that V is strictly increasing. Hence, h is strictly increasing and invertible as a continuous bijection of $[0, \frac{1}{2}]$ onto $[0, \frac{V(\frac{1}{2})}{V(1)}]$. Now, Define $\underline{\delta}$ by $h^{-1}(1-\underline{\delta}\lambda) = \frac{1}{2}$ or $\frac{V(\frac{1}{2})}{V(1)} = 1 - \underline{\delta}\lambda$. Clearly, $1 - \underline{\delta}\lambda \in [0, \frac{1}{2}]$ implies $\underline{\delta} < \frac{1}{\lambda}$.

Case 1: $\delta \in [0, \underline{\delta}]$

$h(\frac{1}{2}) = \frac{V(\frac{1}{2})}{V(1)} = 1 - \underline{\delta}\lambda$ implies that for all $\delta \in [0, \underline{\delta}]$ a politician in power will always choose $\theta = 1$, even when he is paid the total available resource $c_P = \frac{1}{2}$ in all periods. Therefore, when $\delta \in [0, \underline{\delta}]$, politicians are extremely impatient and would not allow for technological growth, no matter how large a fraction of total resources they receive. In this case, it is optimal for citizens to set $c_P = 0$ in all periods. Hence, when $\delta \in [0, \underline{\delta}]$, $c^*_P = 0$ and $\theta^* = 1$ in all periods: the economy never grows.

Case 2: $\delta \in (\underline{\delta}, 1/\lambda)$

$\delta \in (\underline{\delta}, 1/\lambda)$ and $h(0) = 0 \Rightarrow h^{-1}(1 - \delta\lambda) \in (0, 1/2)$.

Now define function g by $g(\beta) = \frac{1-\beta\lambda}{1-\beta}$ for all $\beta \in [0, 1/\lambda)$. g is strictly decreasing with values in $(0, 1]$ for $\beta \in [0, 1/\lambda)$. Because g is also continuous, it is invertible. Now, let's define $\beta^*(\delta) = g^{-1}(\frac{U[\frac{1}{2}-h^{-1}(1-\delta\lambda)]}{U(1/2)})$. Clearly, $\beta^*(\delta) \in (g^{-1}(1), g^{-1}(0)) = (0, \frac{1}{\lambda})$. Note that $U[\frac{1}{2} - h^{-1}(1-\delta\lambda)] \geq \frac{1-\beta\lambda}{1-\beta}$ for all $\beta \in [\beta^*(\delta), 1/\lambda)$, while $U[\frac{1}{2} - h^{-1}(1-\delta\lambda)] < \frac{1-\beta\lambda}{1-\beta}$ for all $\beta \in [0, \beta^*(\delta))$. This implies, $\frac{U[\frac{1}{2}-h^{-1}(1-\delta\lambda)]}{1-\beta\lambda} > \frac{U[1/2]}{1-\beta}$ $\forall$ $\beta \in (\beta^*(\delta), 1/\lambda)$ and $\frac{U[\frac{1}{2}-h^{-1}(1-\delta\lambda)]}{1-\beta\lambda} \leq \frac{U[1/2]}{1-\beta}$ $\forall$ $\beta \in [0, \beta^*(\delta)]$. Therefore, if $\delta \in (\underline{\delta}, 1/\lambda)$ and $\beta \in (\beta^*(\delta), 1/\lambda)$, then $c_P^* = h^{-1}(1-\delta\lambda) \in (0, 1/2)$ and $\theta^* = 0$ (the economy always grows).

If instead $\delta \in (\underline{\delta}, 1/\lambda)$ and $\beta \in [0, \beta^*(\delta)]$, then $\frac{U[\frac{1}{2}-h^{-1}(1-\delta\lambda)]}{1-\beta} \leq \frac{U[1/2]}{1-\beta\lambda}$: the representative citizen fails to achieve his/her outside option value $\frac{U[1/2]}{1-\beta}$ by giving the politician in power just enough to satisfy the politician's sustainability constraint. Now, recall that $h^{-1}(1 - \delta\lambda) \in (0, 1/2)$ is the unique stationary payment to the politician which makes the sustainability constraint hold with equality. It follows that in this case, for any scheme with a stationary and sustainable payment to the politician, the lifetime utility of the representative citizen will always be strictly less than $\frac{U[1/2]}{1-\beta}$. Therefore, if $\delta \in (\underline{\delta}, 1/\lambda)$ and $\beta \in [0, \beta^*(\delta)]$, then $c_P^* = 0$ and $\theta^* = 1$ (the economy never grows).

■

Corollary 7 *Suppose politicians and citizens have common discount factor ρ. Then, there exists $\tilde{\rho}^{\xi=0}$ such that the economy grows iff $\rho > \tilde{\rho}^{\xi=0}$.*

Proof.

For an economy with common discount factor ρ for citizens and politicians, the theorem states that there exists $\underline{\rho} < \frac{1}{\lambda}$ such that if $\rho \leq \underline{\rho}$, then the economy stagnates. Now, suppose $\rho > \underline{\rho}$. Recall that $\frac{U[\frac{1}{2}-h^{-1}(1-\rho\lambda)]}{U[1/2]}$ is continuous and strictly decreasing in ρ while $\frac{1-\rho\lambda}{1-\rho}$ is continuous and strictly decreasing in ρ. Moreover, $\frac{U[\frac{1}{2}-h^{-1}(1-\underline{\rho}\lambda)]}{U[1/2]} = 0 < \frac{1-\underline{\rho}\lambda}{1-\underline{\rho}}$, and $\frac{U[\frac{1}{2}-h^{-1}(1-\frac{1}{\lambda}\lambda)]}{U[1/2]} = 1 > \frac{1-\frac{1}{\lambda}\lambda}{1-\frac{1}{\lambda}}$. Therefore, there exists a unique $\tilde{\rho}^{\xi=0} \in (\underline{\rho}, \frac{1}{\lambda})$ such that $\frac{U[\frac{1}{2}-h^{-1}(1-\tilde{\rho}^{\xi=0}\lambda)]}{1-\tilde{\rho}^{\xi=0}\lambda} = \frac{U[1/2]}{1-\tilde{\rho}^{\xi=0}}$. Given $\rho > \underline{\rho}$, the theorem implies that the economy grows iff $\rho > \tilde{\rho}^{\xi=0}$. Therefore, for any $\rho \in [0, \frac{1}{\lambda})$, the economy grows iff $\rho > \max\{\underline{\rho}, \tilde{\rho}^{\xi=0}\} = \tilde{\rho}^{\xi=0}$. That is , for any $\rho \in [0, \frac{1}{\lambda})$, the economy grows iff $\rho > \tilde{\rho}^{\xi=0}$.

■

2 Coups d'Etat and Effective Discounting

This section analyzes the impact of political instability on economic growth. Assumptions made on preferences and technology in the previous section are maintained . However, the economy considered in this section differs from the previous section's economy in two respects. First, citizens and politicians now have a common discount factor ρ. Second, citizens do no longer have full control over the choice of the future period politician in power. Specifically, it is assumed that at the end of every period, before the representative citizen chooses the future period politician, a coup d'etat might occur with probability c. When a coup d'etat occurs, the representative citizen loses his voting right for the period and the politician in power is replaced with a different politician randomly chosen by nature. It is also assumed that both the representative citizen and nature may not choose a politician previously overthrown by a coup d'etat as the future period politician.

The timing is as follows. The economy starts date 0 with technological frontier S_0 and politician ι_0 in power. After receiving endowments of the consumption and investment goods, the representative citizen gives politician ι_0 a fraction of the endowment of the consumption good. Politician ι_0 then chooses whether to consume the investment good or to process it for growth. Thereafter, the representative citizen and the politician in power consume their respective allocations. Nature then draws ω in $\{0, 1\}$, where $\omega = 1$ is the event of a coup d'etat which has probability c and $\omega = 0$ is the complementary event. If $\omega = 0$, then the representative citizen chooses the next period politician in $J \cup \{\varnothing\}$ and a new period starts. If $\omega = 1$, then nature draws a new politician in $J - \{\iota_0\}$ and a new period starts with the politician drawn by nature as the politician in power. At the end of any future period t during which a coup d'etat has not occurred, the representative citizen chooses politician ι_{t+1} in $(J - \Pi_0^t) \cup \{\varnothing\}$, where Π_0^t is the collection of politicians overthrown by a coup d'etat from date 0 to date t. At the end of any period t during which a coup d'etat has occurred, nature draws politician ι_{t+1} from $J - \Pi_0^t$. The timing for all future periods is similar to that of period 0.

2.1 Relevant Histories, Strategies and Equilibrium Concept

As before, let $h_{c,1}^t$, $h_{c,2}^t$, h_P^t denote the date t histories available to the representative citizen in the first stage, to the politician in power in the second stage and to the representative citizen in the third stage of the period t game. Histories evolve as follows: $h_{c,1}^0 = \{\iota_0, S_0\}$, $h_P^0 = \{h_{c,1}^0, c_0^{HH}, c_0^P\}$, $h_{c,2}^0 = \{h_P^0, \theta_0, S_1\}$, $h_{c,1}^1 = \{h_{c,2}^0, \iota_1\}$ $h_{c,2}^1 = \{h_P^1, \theta_1, S_2, \omega_1\}$ and so on...

The difference between these histories and those in section 1 is that every period, the information set of the stage 3 representative citizen is now augmented with ω. As in section 1, let $H_{c,1}^t|h^t$, $H_{c,2}^t|h^t$ and $H_P^t|h^t$ denote the set of histories that follow some history h^t. Also let $\sigma_{c,1}|h^t, \sigma_P|h^t, \sigma_{c,2}|h^t$ define strategies for the representative citizen in stage 1, the politician in power in stage 2 and the representative citizen in stage 3 of the period game. Let $\Sigma_{c,1}|h^t, \Sigma_P|h^t$ and $\Sigma_{c,2}|h^t$ denote the sets of all such strategies.

Define $\omega_{-1} = 0$ and $\omega_s^t = \{\omega_s, ...\omega_t\}$. Let h^t denote some history ending with states ι_t, S_t and ω_{-1}^{t-1}. Let $\sigma|h^t \equiv (\sigma_{c,1}|h^t, \sigma_P|h^t, \sigma_{c,2}|h^t)$ be a strategy profile which induces the sequence of allocations $\mathcal{A}_t \equiv \{c_{t+\tau}^{HH}(.), c_{t+\tau}^P(.), \theta_{t+\tau}(.), \iota_{t+1+\tau}(.)\}_{\tau=0}^{\infty}$. As before, for any history $\omega_{-1}^{t+\tau-1}$, the law of motion of the technological frontier is: $S_t(\omega_{-1}^{t-2}) \equiv S_t$ and $S_{t+\tau+1}(\omega_{-1}^{t+\tau-1}) = S_{t+\tau}(\omega_{-1}^{t+\tau-2}) + 1 - \theta_{t+\tau}(\omega_{-1}^{t+\tau-1})$.

Define

$$\kappa(\omega_{t-1}^{t+\tau-1}) \equiv \inf\{t \leq s \leq t+\tau-1 : \omega_s = 1\}.$$

Payoffs from $\sigma|h^t$ to the representative citizen and to politician in power ι_t are respectively given by $\Phi_c(\sigma|(h^t, S_t, \iota_t, \omega_{-1}^{t-1})) \equiv \Gamma_c(\mathcal{A}_t)$ and $\Phi_{\iota_t}(\sigma|(h_P^t, S_t, \iota_t, \omega_{-1}^{t-1})) \equiv \tilde{\Gamma}_P(\mathcal{A}_t)$, where

$$\Gamma_c(\mathcal{A}_t) \equiv \sum_{\tau=0}^{\infty} \sum_{\omega_{t-1}^{t+\tau-1}} \rho^\tau \lambda^{S_{t+\tau}(\omega_{-1}^{t+\tau-2})} U[c_{t+\tau}^{HH}(\omega_{-1}^{t+\tau-1})] * \text{ prob } (\omega_{t-1}^{t+\tau-1})$$

$$\tilde{\Gamma}_P(\mathcal{A}_t) \equiv \sum_{\tau=0}^{\infty} \sum_{\omega_{t-1}^{t+\tau-1} | \kappa(\omega_{t-1}^{t+\tau-1}) > t+\tau-1} \rho^\tau \lambda^{S_{t+\tau}(\omega_{-1}^{t+\tau-2})} V[c_{t+\tau}^{P}(\omega_{-1}^{t+\tau-1}) + \frac{\theta_{t+\tau}(\omega_{-1}^{t+\tau-1})}{2}] * \text{prob } (\omega_{t-1}^{t+\tau-1})$$

$$*\mathbf{1}_{\{\iota_{t+\tau(\omega_{t-1}^{t+\tau-1})} = \iota_t\}}.$$

In the above formula, payoff-relevant histories for a politician in power are histories during which he is not overthrown by a coup d'etat. When a politician is not overthrown by a coup d'etat, he gets to consume only if the strategy profile calls for the representative citizen to keep him in power. This is reflected in $\mathbf{1}_{\{\iota_{t+\tau+1(\omega_{t-1}^{t+\tau-1})} = \iota_t\}}$ which is an indicator function that takes on value 1 if the strategy profile calls for the representative citizen to choose politician ι_t as their period $t+\tau$ politician in power given history $\omega_{t-1}^{t+\tau-1}$. Clearly, if $\omega_{t+\tau-1} = 1$ for some $\tau \geq 1$, then $\mathbf{1}_{\{\iota_{t+\tau(\omega_{t-1}^{t+\tau-1})} = \iota_t\}} = 0$ since the representative citizen does not get to choose the next period politician at the end of a period during which a coup d'etat has occurred.

Definition 8 *A Strategy profile $\sigma|\{S_0, \iota_0\}$ is a subgame perfect equilibrium (SPE) of the game if $\forall h^t$ with states $\{S_t, \iota_t, \omega_0^{t-1}\}$, the induced continuation strategies satisfy:*

$$\Phi_c(\sigma|(h^t, S_t, \iota_t, \omega_{-1}^{t-1})) \geq \Phi_c(\gamma, \sigma_P|(h^t, S_t, \iota_t, \omega_{-1}^{t-1}), \sigma_{c,2}|(h^t, S_t, \iota_t, \omega_{-1}^{t-1})), \;\; \forall \gamma \in \Sigma_{c,1}|h^t \tag{8a}$$
$$\Phi_{\iota_t}(\sigma|(h^t, S_t, \iota_t, \omega_{-1}^{t-1})) \geq \Phi_P(\sigma_{c,1}|(h^t, S_t, \iota_t, \omega_{-1}^{t-1}), \gamma, \sigma_{c,2}|(h^t, S_t, \iota_t, \omega_{-1}^{t-1})), \;\; \forall \gamma \in \Sigma_P|h^t \tag{8b}$$
$$\Phi_c(\sigma|h^t, (S_t, \iota_t, \omega_{-1}^{t-1})) \geq \Phi_{c,2}(\sigma_{c,1}|(h^t, S_t, \iota_t, \omega_{-1}^{t-1}), \sigma_P|(h^t, S_t, \iota_t, \omega_{-1}^{t-1}), \gamma), \;\; \forall \gamma \in \Sigma_{c,2}|h^t. \tag{8c}$$

Definition 9 *Given some history ω_{-1}^{t-2}, a sequence of allocations*

$\{c_{t+\tau}^{HH}(.), c_{t+\tau}^{P}(.), \theta_{t+\tau}(.), \iota_{t+1+\tau}(.)\}_{\tau=0}^{\infty}$ *is feasible if* $\forall \tau \geq 0, \ \forall \omega_{t-1}^{t+\tau-1}$, *it satisfies:*

$$\{c_{t+\tau}^{HH}(\omega_{-1}^{t+\tau-1}), c_{t+\tau}^{P}(\omega_{-1}^{t+\tau-1})\} \in \Lambda, \ \textit{where } \Lambda \textit{ defined by equation (1)}$$
$$\theta_{t+\tau}(\omega_{-1}^{t+\tau-1}) \in \{0;1\}$$
$$\iota_{t+1+\tau}(\omega_{-1}^{t+\tau}) \in \begin{cases} (J - \Pi_t^{t+\tau}(\omega_{t-1}^{t+\tau})) \cup \{\varnothing\} \ \textit{if } \omega_{t+\tau} = 0 \\ J - \Pi_t^{t+\tau}(\omega_{t-1}^{t+\tau}) \ \textit{if } \omega_{t+\tau} = 1, \end{cases}$$

where $\Pi_t^{t+\tau}(\omega_{t-1}^{t+\tau}) = \{\iota_{t+s+1}(\omega_{t-1}^{t+s}) \ \textit{if } \omega_{t+s} = 1 \ \textit{for } s \in \{0, \ldots, \tau\}\}$. *That is, if history* $\omega_{t-1}^{t+\tau}$ *occurs, then, according to the sequence of allocations,* $\Pi_t^{t+\tau}(\omega_{t-1}^{t+\tau})$ *will be the collection of politicians overthrown by a coup d'etat between between dates* t *and* $t+\tau$.

An argument analogue to the one used to prove lemma 3 implies that a feasible sequence of allocations $\mathcal{A}_0 \equiv \{c_\tau^{HH}(.), c_\tau^{P}(.), \theta_\tau(.), \iota_\tau(.)\}_{\tau=0}^{\infty}$ which induces continuation sequences $\{\mathcal{A}_t, t \geq 0\}$ is an SPE sequence iff $\forall t \geq 0, \forall \omega_{t-1}^{t-1}$,

$$\Gamma_c(\mathcal{A}_t) \geq \frac{\lambda^{S_t(\omega_{-1}^{t-2})}}{1-\rho} U(1/2), \tag{10}$$

and

$$\Gamma_P(\mathcal{A}_t) \geq \lambda^{S_t(\omega_{-1}^{t-2})} V[c_t^P(\omega_{-1}^{t-1}) + \frac{1}{2}]. \tag{11}$$

Also, as in lemma 4, it is easy to check that if an SPE sequence involves a replacement of the initial politician in this economy, then there is another SPE sequence with no replacement that achieves the same payoff to the representative citizen.

Now, define

$$\Gamma_P(\mathcal{A}_t) \equiv \sum_{\tau=0}^{\infty} \sum_{\omega_{t-1}^{t+\tau-1} | \kappa(\omega_{t-1}^{t+\tau-1}) > t+\tau-1} \rho^\tau \lambda^{S_{t+\tau}(\omega_{-1}^{t+\tau-2})} V[c_{t+\tau}^P(\omega_{-1}^{t+\tau-1}) + \frac{\theta_{t+\tau}(\omega_{-1}^{t+\tau-1})}{2}] \ {*}\text{prob } (\omega_{t-1}^{t+\tau-1}).$$

For a sequence of allocations $\mathcal{A}_0 \equiv \{c_\tau^{HH}(.), c_\tau^{P}(.), \theta_\tau(.), \iota_{1+\tau}(.)\}_{\tau=0}^{\infty}$, let $\mathcal{A}_t$ denote the natural continuation of $\mathcal{A}_0$ for any $t \geq 0$. Then, for the class of economies of interest, the best SPE problem is characterized as:

$$\max_{\mathcal{A}_0 \text{ feasible}} \Gamma_c(\mathcal{A}_0)$$

s.t

$$\Gamma_c(\mathcal{A}_t) \geq \frac{\lambda^{S_t(\omega_{-1}^{t-2})}}{1-\rho}U(1/2) \qquad ,\forall t \geq 0,\ \forall \omega_{t-1}^{t-1} \tag{12a}$$

$$\Gamma_P(\mathcal{A}_t) \geq \lambda^{S_t(\omega_{-1}^{t-2})}V[c_t^P(\omega_{-1}^{t-1}) + \frac{1}{2}],\ \forall t \geq 0,\ \forall \omega_{t-1}^{t-1}. \tag{12b}$$

The objective function of the above problem is homogenous of degree 0 in λ^{S_0}. Moreover, for any period t, λ^{S_t} may be simplified from both sides of the constraint inequalities. It is also easy to check that at any date and given any history, the best SPE problem has the following recursive formulation:

$$\lambda^S \Psi_{HH} = \quad \max U(c^{HH}) + \rho\lambda^{1-\theta} * \lambda^S \Psi_{HH}$$

s.t

$$U(c^{HH}) + \rho\lambda^{1-\theta} * \lambda^S \Psi_{HH} \geq \lambda^S \frac{U(\frac{1}{2})}{1-\rho} \tag{13a}$$

$$\lambda^S \Psi_P \equiv V[c^P + \frac{\theta}{2}] + \rho\lambda^{1-\theta}\lambda^S \Psi_P * \ (1-c) \geq \lambda^S V[c^P + \frac{1}{2}]. \tag{13b}$$

Now, define

$$\phi^{**} = \max_{c_P \in \mathbb{R}\ \theta \in \{0;1\}} \frac{U(\frac{1}{2} - c_P)}{1-\rho\lambda^{1-\theta}} \text{ s.t } \frac{V[c_P + \frac{\theta}{2}]}{1-\rho(1-c)\lambda^{1-\theta}} \geq V[c_P + \frac{1}{2}]\ ,$$

and let argmax ϕ' denote $\{c_{HH}^*, c_P^*, \theta^*\}$ such that $\phi^{**} = \frac{U(\frac{1}{2} - c_P^*)}{1-\rho\lambda^{1-\theta^*}}$ and $c_{HH}^* = \frac{1}{2} - c_P^*$.
Then, the best SPE problem above has a stationary solution $\{c_{HH}^*, c_P^*, \theta^*\}$ defined by:

$$\{c_{HH}^*, c_P^*, \theta^*\} = \begin{cases} \text{argmax } \phi' & \text{if } \Phi^{**} \geq \frac{U(1/2)}{1-\rho} \\ \{1/2, 0, 1\} & \text{if } \Phi^{**} < \frac{U(1/2)}{1-\rho}. \end{cases}$$

The best SPE problem of this economy is therefore identical to that of the economy of the previous section for $\delta = \rho(1-c)$ and $\beta = \rho$. That is, introducing the possibility of a coup d'etat scales the effective discount factor of politicians down by the probability of the event that a coup d'etat does not occur. It follows that if citizens and politicians are sufficiently patient to allow for growth in the basic setup with no political instability, then growth will occurs in this setup iff the probability of a coup d'etat is not too large. This is stated in the next theorem.

Theorem 10 *Suppose $\lambda > 1$. Let $\tilde{\rho}^{\xi=0}$ be defined as in corollary 7. If $\rho \in (\tilde{\rho}^{\xi=0}, \frac{1}{\lambda})$, then, there exists $c^* \in [0,1]$ such that the economy grows iff the probability of coup d'etat c is such that $c < c^*$. If $\rho \in [0, \tilde{\rho}^{\xi=0}]$, then the economy does not grow for any value of c in $[0,1]$.*

Proof. First, suppose $\rho \in (\tilde{\rho}^{\xi=0}, \frac{1}{\lambda})$. Define $\overline{c}$ by $\frac{V(\frac{1}{2})}{V(1)} = 1 - \rho(1-\overline{c})\lambda$. Then, $\frac{V(\frac{1}{2})}{V(1)} < 1 \Rightarrow \rho(1-\overline{c})\lambda > 0 \Rightarrow \overline{c} < 1$.

First suppose $c > \bar{c}$. Then, $\frac{V(\frac{1}{2})}{V(1)} < 1 - \rho(1-c)\lambda$ and it follows that in this case, the politician would not let growth occur even if he was offered the maximal resources available. It is therefore optimal for citizens to set $c_P^* = 0$ and the economy necessarily stagnates in this case.

Now, suppose $c \leq \bar{c}$. It follows that $\frac{V(\frac{1}{2})}{V(1)} \geq 1 - \rho(1-c)\lambda$ while $0 = \frac{V(0)}{V(\frac{1}{2})} < 1 - \rho(1-c)\lambda$. Hence, given $c \leq \bar{c}$, $h^{-1}(1-\rho(1-c)\lambda) \in (0, \frac{1}{2}]$, where h is the function defined by equation (7). Moreover, by the monotonicity argument laid out in section 1, $\frac{U[\frac{1}{2} - h^{-1}(1-\tilde{\rho}^{\xi=0}\lambda)]}{1-\tilde{\rho}^{\xi=0}\lambda} = \frac{U[1/2]}{1-\tilde{\rho}^{\xi=0}}$ and $\rho \geq \tilde{\rho}^{\xi=0}$ imply $\frac{U[\frac{1}{2} - h^{-1}(1-\rho\lambda)]}{1-\rho\lambda} \geq \frac{U[1/2]}{1-\rho}$ or $\frac{U[\frac{1}{2} - h^{-1}(1-\rho(1-c)\lambda)]}{1-\rho\lambda} \geq \frac{U[1/2]}{1-\rho}$ for $c = 0$. However, as argued above, $h^{-1}(1-\rho(1-\bar{c})\lambda) = \frac{1}{2}$ and therefore $\frac{U[\frac{1}{2} - h^{-1}(1-\rho(1-\bar{c})\lambda)]}{1-\rho\lambda} < \frac{U[1/2]}{1-\rho}$. Given that $h^{-1}(1-\rho(1-c)$ is continuous and strictly increasing in c for fixed ρ, it follows from the continuous value theorem that there exits $c^* \in [0, \bar{c})$ such that $\frac{U[\frac{1}{2} - h^{-1}(1-\rho(1-c)\lambda)]}{1-\rho\lambda} > \frac{U[1/2]}{1-\rho}$ iff $c < c^*$. Hence for fixed ρ, the economy grows iff $c < \min\{\bar{c}, c^*\} = c^*$. This ends the proof of the theorem.

Now, suppose $\rho \in [0, \tilde{\rho}^{\xi=0}]$. Then, $\frac{U[\frac{1}{2} - h^{-1}(1-\rho\lambda)]}{1-\rho\lambda} \leq \frac{U[1/2]}{1-\rho}$ and therefore, $\frac{U[\frac{1}{2} - h^{-1}(1-\rho(1-c)\lambda)]}{1-\rho\lambda} \leq \frac{U[1/2]}{1-\rho}, \forall c \in [0,1]$. It follows that in this case, the economy does not grow for any probability c.

■

3 Dictatorship and Economic Growth

This section analyzes the impact of dictatorship on economic growth. Assumptions on preferences and technology made in section 1 are maintained. As in section 2, citizens and politicians are assumed to have a common discount factor ρ. Also as in section 2, citizens do not have full control over the choice of the politician in power. However, in this section, the loss of voting rights is caused by a different friction. In section 2, the loss of voting rights was temporary and succeeded the occurrence of a coup d'etat. In this section, the loss of voting rights is permanent and occurs after a politician in power has become a dictator. Specifically, this section assumes that at the end of every period, before citizens vote, the politician in power draws a dictatorship ticket $d \in \{0, 1\}$. If the politician draws $d = 1$, then the representative citizen is not allowed to choose the next period's politician. In this case, the politician who made the draw stays in power forever. If the politician draws $d = 0$, then the representative citizen chooses the next period's politician in power and a new period will start. A politician who has drawn $d = 1$ is referred to as a dictator. The economy is called a dictatorship or a dictatorial economy when the politician in power is a dictator. An economy with $\xi = 0$ is referred to as a democracy or a democratic economy. Conditional on not being a dictator, the probability that a politician draws $d = 1$ is assumed equal to $\xi \in [0, 1]$ in all periods. If $d_t = 1$ in period t, then with probability 1, $d_{t+s} = 1, \forall s \geq 0$.

The timing of the game in a non-dictatorial economy is as follows. The economy starts date 0 with politician ι_0. First, the representative citizen receives endowments of the consumption and investment goods and decides on the allocation of the consumption good between the politician in power and citizens. The politician in power is then entrusted with the investment good and chooses whether to consume it or to process it to shift the technological frontier. Afterward, the citizens and the politician in power consume their respective allocations. The politician then draws d in $\{0,1\}$. If $d=0$, then the representative citizen chooses the next period's politician and a new period starts. If $d=1$, then the economy becomes a dictatorship and a new period starts. The timing in a dictatorship is the following: the representative citizen receives the endowments and allocates the consumption good between the dictator and the citizens. The dictator then decides whether or not to shift the technological frontier. Consumption occurs and a new period starts.

3.1 Relevant Histories, Strategies and Equilibrium Concept

As before, let $h^t_{c,1}$, $h^t_{c,2}$, h^t_P denote date t histories available to the representative citizen in the first stage, to the politician in power in the second stage and to the representative citizen in the third stage of the period t game. The evolution of histories is conditioned on the realization of the dictatorship ticket in the following fashion:

$h^0_{c,1}=\{\iota_0,S_0\},\ h^0_P=\{h^0_{c,1},c^{HH}_0,c^P_0\},\ h^0_{c,2}=\{h^0_P,\theta_0,S_1,d_0\}$
$h^1_{c,1}|(d_0=0)=\{h^0_{c,2},\iota_1\},\ h^1_P|(d_0=0)=\{h^1_{c,1},c^{HH}_1,c^P_1\},\ h^1_{c,2}|(d_0=0)=\{h^1_P,\theta_1,S_2,d_1\}$
$h^1_{c,1}|(d_0=1)=\{h^0_{c,2}\},\ h^1_P|(d_0=1)=\{h^1_{c,1},c^{HH}_1,c^P_1\}$
$h^1_{c,2}|(d_0=1)=\{h^1_P,\theta_1,S_2,d_1\}$
$h^2_{c,1}|(d_0=0\text{ and }d_1=0)=\{h^1_{c,2},\iota_2\},\ \ h^1_P|(d_0=0\text{ and }d_1=0)=\{h^1_{c,1},c^{HH}_2,c^P_2\},$
$h^1_{c,2}|(d_0=0\text{ and }d_1=0)=\{h^1_P,\theta_2,S_3,d_2\}$
$h^2_{c,1}|(d_0=0\text{ and }d_1=1)=\{h^1_{c,2}\},\ \ h^1_P|(d_0=0\text{ and }d_1=1)=\{h^1_{c,1},c^{HH}_2,c^P_2\},$
$h^1_{c,2}|(d_0=0\text{ and }d_1=1)=\{h^1_P\theta_2,S_3\}$
$h^2_{c,1}|(d_0=1)=\{h^1_{c,2}\},\ \ h^1_P|(d_0=1)=\{h^1_{c,1},c^{HH}_2,c^P_2\},h^1_{c,2}|(d_0=1)=\{h^1_P,\theta_2,S_3\}$, and so on ...

More generally, let's define $d_{-1}=0$ and $d^t=\{d_0,...d_t\}$ and

$$\kappa(d^t)=\ \inf\ \{-1\le s\le t:d_s=1\}. \tag{15}$$

Also, let's define by $\pi(d^t|d^{t-1})$ the probability that the economy reaches $d^t\equiv(d^{t-1},d_t)$ given current state d^{t-1}.

Then,

$$\pi(d^t|d^{t-1})=\begin{cases}1\text{ if }d_t=1\text{ and }0\text{ if }d_t=0, & \text{given }\ \kappa(d^{t-1})\le t-1\\ \xi\text{ if }d_t=1\text{ and }1-\xi\text{ if }d_t=0, & \text{given }\ \kappa(d^{t-1})>t-1.\end{cases}$$

Relevant histories in a democratic economy evolve as:

$$\begin{cases} h^t_{c,1}|(d^{t-1}|\kappa(d^{t-1} > t-1)) = \{\iota_t, S_t\} \\ h^t_P|(d^{t-1}|\kappa(d^{t-1} > t-1)) = \{h^t_{c,1}|(d^{t-1}|\kappa(d^{t-1} > t-1)), c^{HH}_t, c^P_t\} \\ h^t_{c,2}|(d^{t-1}|\kappa(d^{t-1} > t-1)) = \{h^t_P|(d^{t-1}|\kappa(d^{t-1} > t-1)), \theta_t, S_{t+1}, d_t\}, \end{cases}$$

while in a dictatorship, relevant histories are given by:

$$\begin{cases} h^t_{c,1}|(d^{t-1}|\kappa(d^{t-1} \leq t-1)) = \{\iota_{\kappa(d^{t-1})}, S_t\}\} \\ h^t_P|(d^{t-1}|\kappa(d^{t-1} \leq t-1)) = \{h^t_{c,1}|(d^{t-1}|\kappa(d^{t-1} \leq t-1)), c^{HH}_t, c^P_t. \end{cases}$$

As in the previous sections, let $H^t_{c,1}|h^t$, $H^t_{c,2}|h^t$ and $H^t_P|h^t$ denote the set of histories that follow some history h^t. Also let $\sigma_{c,1}|h^t, \sigma_P|h^t, \sigma_{c,2}|h^t$ define strategies for the representative citizen in stage 1, the politician in power in stage 2 and the representative citizen in stage 3 of the period t game. Let $\Sigma_{c,1}|h^t, \Sigma_P|h^t$ and $\Sigma_{c,2}|h^t$ denote the sets of all such strategies.

For any history h^t and strategies $\sigma_{c,1}|h^t, \sigma_P|h^t, \sigma_{c,2}|h^t$, let $\sigma|h^t \equiv (\sigma_{c,1}|h^t, \sigma_P|h^t, \sigma_{c,2}|h^t)$ denote the corresponding strategy profile. Let $h^t|(S_t, \iota_t, d^{t-1}|\kappa(d^{t-1}) < t-1)$ denote a history h^t which ends with technological frontier S_t and politician in power ι_t who is a dictator. Similarly, let $h^t|(S_t, \iota_t, d^{t-1}|\kappa(d^{t-1}) \geq t-1)$ denote a history h^t which ends with technological frontier S_t and politician in power ι_t who is not a dictator. Because dictatorship is an absorbing state , a strategy profile $\sigma|(h^t, S_t, \iota_t, d^{t-1}|\kappa(d^{t-1}) \leq t-1)$ induces a sequence of allocations $\{c^{HH}_{t+\tau}, c^P_{t+\tau}, \theta_{t+\tau}, \iota_{t+\tau}\}^{\infty}_{\tau=0}$ which is not a function of future draws of the dictatorship tickets. A strategy profile $\sigma|(h^t, S_t, \iota_t, d^{t-1}|\kappa(d^{t-1}) > t-1)$ induces a sequence of allocations $\{c^{HH}_{t+\tau}(.), c^P_{t+\tau}(.), \theta_{t+\tau}(.), \iota_{t+\tau}(.)\}^{\infty}_{\tau=0}$, where $\{c^{HH}_{t+\tau}(.), c^P_{t+\tau}(.), \theta_{t+\tau}(.), \iota_{t+\tau}(.)$ are functions of histories $d^{t+\tau}$. Define $d_{-1} = 0$ and:

$$\mathcal{A}(d^{t+\tau-1}) \equiv (c^{HH}_{t+\tau}(d^{t+\tau-1}), c^P_{t+\tau}(d^{t+\tau-1}), \theta_{t+\tau}(d^{t+\tau-1})) \tag{16a}$$

$$\Omega^{HH}(\mathcal{A}(d^{t+\tau-1})) \equiv \lambda^{\tau-\theta_t(d^{t+\tau-1})-\theta_{t+1}(d^{t-1})-\ldots-\theta_{t+\tau-1}(d^{t+\tau-2})} U[c^{HH}_{t+\tau}(d^{t+\tau-1})] \tag{16b}$$

$$\Omega^P(\mathcal{A}(d^{t+\tau-1})) \equiv \lambda^{\tau-\theta_t(d^{t-1})-\theta_{t+1}-\ldots-\theta_{t+\tau-1}(d^{t+\tau-2})} V[c^P_{t+\tau}(d^{t+\tau-1}) + \frac{\theta_{t+\tau}(d^{t+\tau-1})}{2}]. \tag{16c}$$

For all $t \geq 0$, I also define:

$$\sigma^D|(S_t, \iota_t, d^{t-1}) = \sigma|(S_t, \iota_t, d^{t-1}|\kappa(d^{t-1}) \leq t-1)$$
$$\sigma^{ND}|(S_t, \iota_t, d^{t-1}) = \sigma|(S_t, \iota_t, d^{t-1}|\kappa(d^{t-1}) > t-1).$$

Then, resulting payoffs to the representative citizen and to politician ι_t in power from a strategy profile $\sigma^D|(S_t, \iota_t, d^{t-1})$ which induces a sequence of allocations

$\{c^{HH}_{t+\tau}, c^P_{t+\tau}, \theta_{t+\tau}, \iota_{t+\tau}\}^\infty_{\tau=0}$ are:

$$\Phi_c(\sigma^D|(S_t,\iota_t,d^{t-1})) = \sum_{\tau=0}^{\infty} \rho^\tau \lambda^{S_{t+\tau}} U[c^{HH}_{t+\tau}] \tag{17}$$

and

$$\Phi_{\iota_t}(\sigma^D|(S_t,\iota_t,d^{t-1})) = \sum_{\tau=0}^{\infty} \rho^\tau \lambda^{S_{t+\tau}} V[c^P_{t+\tau} + \frac{\theta_{t+\tau}}{2}]. \tag{18}$$

Similarly, resulting payoffs to the representative citizen and to the politician in power from a strategy profile strategy profile $\sigma^{ND}|(S_t,\iota_t,d^{t-1})$ with induced sequence of allocations $\{\mathcal{A}(d^{t+\tau-1})\}^\infty_{\tau=0}$ are given by:

$$\Phi_{HH}(\sigma^{ND}|(S_t,\iota_t,d^{t-1})) = \lambda^{S_t} \sum_{\tau=0}^{\infty} \sum_{d^{t+\tau-1} \succeq d^{t-1}} \Omega^{HH}(\mathcal{A}(d^{t+\tau-1}))\pi(d^{t+\tau-1}|d^{t-1}) \tag{19}$$

and

$$\Phi_{\iota_t}(\sigma^{ND}|(S_t,\iota_t,d^{t-1})) = \lambda^{S_t} \sum_{\tau=0}^{\infty} \sum_{d^{t+\tau-1} \succeq d^{t-1}} \Omega^{P}(\mathcal{A}(d^{t+\tau-1}))\pi(d^{t+\tau-1}|d^{t-1}) * \mathbf{1}_{\{\iota_{t+1}(d^{t+\tau})=\iota_t\}}. \tag{20}$$

For a politician in power in a non-dictatorial economy, $\mathbf{1}_{\{\iota_{t+1}(d^{t+\tau})=\iota_t\}}$ in the expression of payoffs to the politician reflects the fact that a politician who has not yet drawn a dictatorship ticket will remain in power only if the strategy profile calls for the representative citizen to keep him in power. Note that $\mathbf{1}_{\{\iota_{t+1}(d^{t+\tau})=\iota_t\}}$ is absent from the expression of payoffs to a dictator given that a dictator is always guaranteed to remain in power.

Definition 11 *A Strategy profile $\sigma|\{S_0,\iota_0\}$ is a subgame perfect equilibrium (SPE) of the game if $\forall h^t$ with states $\{S_t,\iota_t,d^{t-1}\}$, the induced continuation strategies satisfy:*

$$\Phi_c(\sigma|h^t) \geq \Phi_c(\gamma, \sigma_P|h^t, \sigma_{c,2}|h^t), \forall \gamma \in \Sigma_{c,1}|h^t \tag{21}$$

$$\Phi_{\iota_t}(\sigma|h^t) \geq \Phi_P(\sigma_{c,1}|h^t, \gamma, \sigma_{c,2}|h^t), \forall \gamma \in \Sigma_P|h^t \tag{22}$$

$$\Phi_{c,2}(\sigma|h^t) \geq \Phi_{c,2}(\sigma_{c,1}|h^t, \sigma_P|h^t, \gamma), \forall \gamma \in \Sigma_{c,2}|h^t. \tag{23}$$

Definition 12 *Given $d_{-1} = 0$, a sequence of allocations $\{c^{HH}_\tau(.), c^P_\tau(.), \theta_\tau(.), \iota_\tau(.)\}^\infty_{\tau=0}$ is fea-*

sible if $\forall \tau \geq 0$. *and* $\forall d^{\tau-1}$, *it satisfies:*

$c_{\tau}^{HH}(d^{\tau-1}) + c_{\tau}^{P}(d^{\tau-1}) = 1/2,$
$\theta_{\tau}(d^{\tau-1}) \in \{0;1\},$
$\iota_{\tau}(d^{\tau-1}) \in J \cup \{\varnothing\}$ *if* $\kappa(d^{\tau-1}) > \tau - 1$ *and* $\iota_{\tau}(d^{\tau-1}) = \iota_{\kappa(d^{\tau-1})}(d^{\kappa(d^{\tau-1})})$ *if* $\kappa(d^{\tau-1}) \leq \tau - 1$,
where $d^{\kappa(d^{\tau-1})}$ *is the truncation of* $d^{\tau-1}$ *at date* $\kappa(d^{\tau-1})$.

The next lemma characterizes SPE sequences by supporting them with trigger strategies that threaten to shift to the worst outcome of the economy whenever a deviation occurs. The worst outcome of this economy is a situation in which the politician in power always receives $c_P = 0$ whether or not he is a dictator, always stops growth by choosing $\theta = 0$ whether or not he is a dictator, and always gets fired whenever not a dictator.

Let's define

$$u_b^* = \frac{U(1/2)}{1-\rho}, \tag{25}$$

and

$$v_b^* = \frac{V(1/2)}{1-\rho}. \tag{26}$$

Lemma 13 *A feasible sequence of allocations* $\{c_{t+\tau}^{HH}(.), c_{t+\tau}^{P}(.), \theta_{t+\tau}(.), \iota_{t+\tau}(.)\}_{\tau=0}^{\infty}$ *from some strategy profile* $\sigma|(S_t, \iota_t, d^{t-1})$ *is an SPE sequence iff* $\forall s \geq 0, \forall d^{t+s-1} \succeq d^{t-1}$ *and* $S_{t+s}(d^{t+s-2}) = S_t + s - \theta_t - - - \theta_{t+s-1}(d^{t+s-2})$,

$$\Phi_c(\sigma^{ND}|(d^{t+s-1}, S_{t+s}(d^{t+s-2}), \iota_{t+s}(d^{t+s-1}))) \geq \lambda^{S_{t+s}(d^{t+s-2})} U_b^* \tag{27a}$$

$$\Phi_c(\sigma^{D}|(d^{t+s-1}, S_{t+s}(d^{t+s-2}), \iota_{t+s}(d^{t+s-1}))) \geq \lambda^{S_{t+s}(d^{t+s-2})} U_b^* \tag{27b}$$

$$\Phi_{\iota_t}(\sigma^{ND}|(d^{t+s-1}, S_{t+s}(d^{t+s-2}), \iota_{t+s}(d^{t+s-1}))) \geq \lambda^{S_{t+s}(d^{t+s-2})} V[c_{t+s}^{P}(d^{t+s-1}) + \frac{1}{2}] + \rho\xi\lambda^{S_{t+s}(d^{t+s-2})} v_b^* \tag{27c}$$

$$\Phi_{\iota_t}(\sigma^{D}|(d^{t+s-1}, S_{t+s}(d^{t+s-2}), \iota_{t+s}(d^{t+s-1}))) \geq \lambda^{S_{t+s}(d^{t+s-2})} V[c_{t+s}^{P}(d^{t+s-1}) + \frac{1}{2}] + \rho\lambda^{S_{t+s}(d^{t+s-2})} v_b^*. \tag{27d}$$

Proof. The proof is similar to that of lemma 3. Let's first show necessity of (27a) to (27d). Suppose $\sigma|(S_t, \iota_t, d^{t-1})$ induces the sequence of allocations $c_{t+\tau}^{P}(.), \theta_{t+\tau}(.), \iota_{t+\tau}(.)\}_{t=0}^{\infty}$. Now, let's suppose that after some date $t+s$ with history d^{t+s-1}, the representative citizen in stage 1 decides to deviate from $\sigma|(S_t, \iota_t, d^{t-1})$ by choosing the allocation $\{\hat{c}_{t+s+\tau}^{HH}(d^{t+s+\tau-1}), \hat{c}_{t+s+\tau}^{P}(d^{t+s+\tau-1})\} = \{1/2, 0\}$ for all $\tau \geq 0$, and $\forall d^{t+s+\tau-1}$. Assuming all other players follow $\sigma|(S_t, \iota_t, d^{t-1})$, the payoff to the representative citizen from this deviation is:

$$\sum_{\tau=0}^{\infty} \rho^{\tau} \sum_{d^{t+\tau+s-1} \succeq d^{t+s-1}} \lambda^{S_{t+s+\tau}(d^{t+s+\tau-1})} \pi(d^{t+s+\tau-1}|d^{t+s-1}) U(1/2)$$

$\geq \lambda^{S_{t+s}(d^{t+s-1})} \frac{U(1/2)}{1-\rho} \equiv \lambda^{S_{t+s}(d^{t+s-1})} u_b^*$. Now because $\sigma|(S_t, \iota_t, d^t)$ is an SPE strategy profile, it holds that $\Phi_c(\sigma^j|(d^{t+s-1}, S_{t+s}(d^{t+s-1}), \iota_{t+s}(d^{t+s-1}))) \geq \lambda^{S_{t+s}(d^{t+s-1})} u_b^*$, $j \in \{ND, D\}$ implying (27a) and (27b). To show necessity of (27c) and (27d), let's suppose a politician ι_{t+s} in power at date $t+s$ following some history d^{t+s-1} decides to deviate by choosing $\hat{\theta}_{t+s+\tau}(d^{t+s+\tau-1}) = 1$ for all $\tau \geq 0$ and for all d^{t+s-1}, conditional on being in power in $t+s+\tau$. The payoff to the politician if he deviates unilaterally in this fashion is:

$$\sum_{\tau=0}^{\infty} \sum_{d^{t+\tau+s-1} \succeq d^{t+s-1}} \rho^{\tau} \lambda^{S_{t+s+\tau}(d^{t+s-1})} V[c_{t+\tau}^P(d^{t+\tau-1}) + \frac{1}{2}] * \pi(d^{t+s+\tau-1}|d^{t+s-1}) * \mathbf{1}_{\{\iota_{t+s+\tau}(d^{t+s+\tau-1}) = \iota_{t+s}\}}$$

$$\geq \lambda^{S_{t+s}(d^{t+s-1})} V[c_{t+s}^P(d^{t+s-1}) + \frac{1}{2}] + \rho \lambda^{S_{t+s}(d^{t+s-1})} v_b^*$$

$$\geq \lambda^{S_{t+s}(d^{t+s-1})} V[c_{t+s}^P(d^{t+s-1}) + \frac{1}{2}] + \rho \xi \lambda^{S_{t+s}(d^{t+s-1})} v_b^*.$$

This proves necessity of (27c) and (27d). To show sufficiency, let's first suppose (27a) to (27d) hold for some sequence of allocations $\{\mathcal{A}(.)\}_{\tau=0}^{\infty} \equiv \{c_{t+\tau}^{HH}(.), c_{t+\tau}^{HH}(.), c_{t+\tau}^P(.), \theta_{t+\tau}(.)\}_{\tau=0}^{\infty}$. It needs to be shown that there exists an SPE that induces this sequence. For this end, let's consider the following trigger strategy: if all players have always followed $\{\mathcal{A}(.)\}_{\tau=0}^{\infty}$, then the player called to play continues to follow; if any player has ever deviated, then the strategy calls to move to the worst outcome: the representative citizen always chooses $c^P = 0$, politicians in power always choose $\theta = 1$, and the representative citizen always fires any politician in power who is not a dictator. Because the right hand sides of (27a) to (27d) represent payoffs from best deviations given this trigger strategy profile, citizens in the first stage and politicians in power never want to unilaterally deviate from the trigger strategy profile. It is then left to check that the representative citizen in third stage does not want to deviate as well. Under the defined trigger strategy, if the representative citizen deviates at date $t+s$ after some history d^{t+s-1} by choosing a politician in power different from what indicated by $\{\mathcal{A}(.)\}_{\tau=0}^{\infty}$, then the citizens will receive payoff $\lambda^{S_{t+s}(d^{t+s-1})} U(c_{t+s}^H(d^{t+s-1})) + \frac{\rho}{1-\rho} \lambda^{S_{t+s}(d^{t+s-1})} U(1/2)$ which by (27c) and (27d) is smaller than the citizens'payoff under $\{\mathcal{A}(.)\}_{\tau=0}^{\infty}$. The representative citizen therefore does not want to deviate from $\{\mathcal{A}(.)\}_{\tau=0}^{\infty}$ in the third stage as well. Hence, the defined trigger strategy profile is an SPE which induces the sequence $\{\mathcal{A}(.)\}_{\tau=0}^{\infty}$. This ends the proof of the lemma.

■

In what follows, best SPE sequences are referred to as SPE-induced allocations that maximize payoffs to the representative citizen. A straightforward argument analogue to the one used to prove lemma 4 implies that citizens do not strictly improve their payoffs by replacing a politician in power. The best SPE problem of this economy will therefore without loss of generality restrict to best SPE sequences which involve no replacement of the initial politician. Let $\mathcal{A}(d^{t+\tau}) \equiv \{c^{HH}_{t+\tau}(d^{t+\tau}), c^{P}_{t+\tau}(d^{t+\tau}), \theta_{t+\tau}(d^{t+\tau})\}$ and let Ω^{HH} and Ω^{P} be given by equations (16c) and (16d). Define:

$$E^i(\{\mathcal{A}(.)\}_{\tau=0}^{\infty}|d^{t-1}) \equiv \sum_{\tau=0}^{\infty} \sum_{d^{t+\tau-1} \succeq d^{t-1}} \Omega^i(\mathcal{A}(d^{t+\tau-1}))\pi(d^{t+\tau-1}|d^{t-1}), i \in \{HH, P\}. \tag{29}$$

The Best SPE problem of this economy is then given by:

$$\max_{\{c^{HH}_{t+\tau}(.), c^{P}_{t+\tau}(.), \theta_{t+\tau}(.)\}_{\tau=0}^{\infty} \text{feasible}} \lambda^{S_t} E^{HH}(\{\mathcal{A}(.)\}_{\tau=0}^{\infty}|d^{t-1}) \text{ s.t. } \forall \geq 0, \forall d^{t+s-1} \succeq d^{t-1}$$

$$\lambda^{S_{t+s}(d^{t+s-2})} E^{HH}(\{\mathcal{A}(.)\}_{\tau=0}^{\infty}|d^{t+s-1}) \geq \lambda^{S_{t+s}(d^{t+s-2})} \frac{U(1/2)}{1-\rho},$$

$$\lambda^{S_{t+s}(d^{t+s-2})} E^{P}(\{\mathcal{A}(.)\}_{\tau=0}^{\infty}|d^{t+s-1}) \geq \lambda^{S_{t+s}(d^{t+s-2})} V[c^{P}_{t+s}(d^{t+s-1}) + \frac{1}{2}] + \rho\xi v_b^*,$$

$$\text{if } \kappa(d^{t+s-1}) > t+s-1$$

$$\lambda^{S_{t+s}(d^{t+s-2})} E^{P}(\{\mathcal{A}(.)\}_{\tau=0}^{\infty}|d^{t+s-1}) \geq \lambda^{S_{t+s}(d^{t+s-2})} V[c^{P}_{t+s}(d^{t+s-1}) + \frac{1}{2}] + \rho v_b^*,$$

$$\text{if } \kappa(d^{t+s-1}) \leq t+s-1,$$

$$S_t \text{ given }, S_{t+\tau+1}(d^{t+\tau-1}) = S_{t+\tau}(d^{t+\tau-2}) + 1 - \theta_{t+\tau}(d^{t+s-1}),$$

which simplifies to:

$$\max_{\{c^{HH}_{t+\tau}(.), c^{P}_{t+\tau}(.), \theta_{t+\tau}(.)\}_{\tau=0}^{\infty} \text{feasible}} E^{HH}(\{\mathcal{A}(.)\}_{\tau=0}^{\infty}|d^{t-1}) \text{ s.t. } \forall \geq 0, \forall d^{t+s-1} \succeq d^{t-1}$$

$$E^{HH}(\{\mathcal{A}(.)\}_{\tau=0}^{\infty}|d^{t+s-1}) \geq \frac{U(1/2)}{1-\rho},$$

$$E^{P}(\{\mathcal{A}(.)\}_{\tau=0}^{\infty}|d^{t+s-1}) \geq V[c^{P}_{t+s}(d^{t+s-1}) + \frac{1}{2}] + \rho\xi v_b^*, \text{ if } \kappa(d^{t+s-1}) > t+s-1$$

$$E^{P}(\{\mathcal{A}(.)\}_{\tau=0}^{\infty}|d^{t+s-1}) \geq V[c^{P}_{t+s}(d^{t+s-1}) + \frac{1}{2}] + \rho v_b^*, \text{if } \kappa(d^{t+s-1}) \leq t+s-1,$$

$$S_t \text{ given }, S_{t+\tau+1}(d^{t+\tau-1}) = S_{t+\tau}(d^{t+\tau-2}) + 1 - \theta_{t+\tau}(d^{t+s-1}).$$

Because dictatorship is an absorbing state for the economy, lemma 13 then implies that at any date t, the best SPE problem of an economy that has become a dictatorship is:

$$\max_{\{c^{HH}_{t+\tau}, c^{P}_{t+\tau}, \theta_{s+\tau}\}_{\tau=0}^{\infty} \text{ feasible}} \sum_{\tau=0}^{\infty} \rho^{\tau} \lambda^{S_{t+\tau}} U[c^{HH}_{t+\tau}] \text{ s.t.}$$
$$\sum_{\tau=0}^{\infty} \rho^{\tau} \lambda^{S_{t+s+\tau}} U[c^{HH}_{t+s+\tau}] \geq \lambda^{S_{t+s}} U^*_b, \forall s \geq 0$$
$$\sum_{\tau=0}^{\infty} \rho^{\tau} \lambda^{S_{t+s+\tau}} V[c^{P}_{t+s+\tau} + \frac{\theta_{t+s+\tau}}{2}] \geq \lambda^{S_{t+s}} V[c^{P}_{t+s} + \frac{1}{2}] + \rho v^*_b, \forall s \geq 0$$
$$S_t \text{ given }, S_{t+\tau+1} = S_{t+\tau} + 1 - \theta_{t+\tau}, \forall \tau \geq 0.$$

By an argument identical to the outline in the proof of lemma 5, the above problem is stationary and reduces to:

$$\Phi^c_D = \max\{u^*_b, \max_{\{c^{HH}_D, c^P_D\}} \frac{U(c^D_{HH})}{1-\rho\lambda} s.t. \frac{V(c^P_D)}{1-\rho\lambda} \geq V[c^P_D + \frac{1}{2}] + \rho v^*_b\}.$$

Let's now define real function h_D by:

$$h^D_\rho(c) = \frac{V(c)}{V[c+\frac{1}{2}] + \rho v^*_b}. \tag{33}$$

For given ρ and λ, $h^D_\rho(0) = 0$ and $h^D_\rho(c)$ converges to 1 as c goes to ∞. Moreover, $h^D_\rho(.)$ is continuous and strictly increasing in c. Hence because for any λ and for any $\rho \in [0, \frac{1}{\lambda})$, $1-\rho\lambda \in (0,1]$, there exists a unique $c(\rho) \in (0, +\infty]$ implicitly defined by

$$h^D(c(\rho)) = 1 - \rho\lambda. \tag{34}$$

It should be noted that at this point no restriction is imposed ρ to guarantee that $c(\rho)$ is smaller than $\frac{1}{2}$.

The following series of lemma establishes several results that will be useful in proving the first theorem of this section.

Lemma 14 *Suppose $\rho \in [0, \frac{1}{\lambda})$. Then, $\frac{V(c)}{1-\rho\lambda} - \rho\frac{V(\frac{1}{2})}{1-\rho}$ is strictly increasing in ρ iff $V(c) > \frac{1}{\lambda}V(\frac{1}{2})\frac{(1-\rho\lambda)^2}{(1-\rho)^2}$.*

Proof. Let $\lambda > 1$ be given. For fixed c , define $l(\rho) = \frac{V(c)}{1-\rho\lambda} - \rho\frac{V(\frac{1}{2})}{1-\rho}$. Then, $l'(\rho) = \frac{\lambda V(c)}{(1-\rho)^2} - \frac{V(\frac{1}{2})}{(1-\rho)^2}$. Therefore, $l'(\rho) > 0$ iff $V(c) > \frac{1}{\lambda}V(\frac{1}{2})\frac{(1-\rho\lambda)^2}{(1-\rho)^2}$. This ends the proof of the lemma. ■

Lemma 15 *Suppose c is such that $V(c) \leq \frac{1}{\lambda}V(\frac{1}{2})\frac{(1-\rho\lambda)^2}{(1-\rho)^2}$. Then, $\frac{V(c)}{1-\rho\lambda} < \frac{V(\frac{1}{1})}{1-\rho}$.*

Proof.

Suppose $V(c) \leq \frac{1}{\lambda} V(\frac{1}{2}) \frac{(1-\rho\lambda)^2}{(1-\rho)^2}$. Then, $V(c) \leq \frac{1}{\lambda} V(\frac{1}{2}) \frac{(1-\rho\lambda)^2}{(1-\rho)^2} < V(\frac{1}{2}) \frac{1-\rho\lambda}{1-\rho}$ since $\frac{1}{\lambda} < 1$ and $\frac{(1-\rho\lambda)^2}{(1-\rho)^2} < 1$, as $\lambda > 1$ and $\rho \in [0, \frac{1}{2})$. This in turn implies $\frac{V(c)}{1-\rho\lambda} < \frac{V(\frac{1}{1})}{1-\rho}$. ■

Lemma 16 *For any $\rho \in [0, \frac{1}{\lambda})$, let $c(\rho)$ be defined by equation (34). Suppose $\rho_2 > \rho_1$. Then, $c_{\rho(1)} > c_{\rho(2)}$.*

Proof. Consider $\rho_2 > \rho_1$ and $c_{\rho(i)}, i \in \{1, 2\}$ defined by $h^D(c(\rho)) = 1 - \rho\lambda$ or $\frac{V(c(\rho_i))}{1-\rho_i\lambda} - \rho_i \frac{V(\frac{1}{2})}{1-\rho_i} = V(c(\rho_i) + \frac{1}{2})$. First observe that this implies $c(\rho_i) > 0, i \in \{1, 2\}$. Therefore, $\frac{V(c(\rho_1))}{1-\rho_1\lambda} - \rho_1 \frac{V(\frac{1}{2})}{1-\rho_1} = V(c(\rho_1) + \frac{1}{2})$ implies $\frac{V(c(\rho_1))}{1-\rho_1\lambda} > \frac{V(\frac{1}{2})}{1-\rho_1}$. Then, lemma 15 implies $V(c) > \frac{1}{\lambda} V(\frac{1}{2}) \frac{(1-\rho_1\lambda)^2}{(1-\rho_1)^2}$. Lemma 14 then implies that $\frac{V(c(\rho_1))}{1-\rho_1\lambda} - \rho_1 \frac{V(\frac{1}{2})}{1-\rho_1} < \frac{V(c(\rho_1))}{1-\rho_2\lambda} - \rho_2 \frac{V(\frac{1}{2})}{1-\rho_2}$, as $\rho_2 > \rho_1$. Hence, $V(c(\rho_1) + \frac{1}{2}) < \frac{V(c(\rho_1))}{1-\rho_2\lambda} - \rho_2 \frac{V(\frac{1}{2})}{1-\rho_2}$, or $\frac{V(c(\rho_1))}{1-\rho_2\lambda} - V(c(\rho_1) + \frac{1}{2}) > \rho_2 \frac{V(\frac{1}{2})}{1-\rho_2}$. Therefore, $\frac{V(c(\rho_2))}{1-\rho_2\lambda} - V(c(\rho_2) + \frac{1}{2}) = \rho_2 \frac{V(\frac{1}{2})}{1-\rho_2}$ implies $c_{\rho(1)} > c_{\rho(2)}$, since from concavity of V, $\frac{V(c)}{1-\rho\lambda} - V(c + \frac{1}{2})$ is strictly increasing in c for any given ρ and λ. ■

Lemma 17 *Let $c(\rho)$ be uniquely defined on $(0, \frac{1}{\lambda})$ by $\frac{V(c(\rho))}{1-\rho\lambda} - \rho \frac{V(\frac{1}{2})}{1-\rho} = V(c(\rho) + \frac{1}{2})$. Then, $c(.)$ is a continuous function of ρ. Moreover, $c((0, \frac{1}{\lambda})) = (0 + \infty)$.*

Proof.

Lemma (16) has established that $c(.)$ is a monotonic function. To show that $c(.)$ is also continuous, it then suffices to show that $\{c(\rho_n)\}$ converges to $c(\rho)$ whenever $\{\rho_n\}$ is a monotonic sequence in $(0, \frac{1}{\lambda})$ that converges to $\rho \in [0, \frac{1}{\lambda}]$.

First consider the case $\rho \in (0, \frac{1}{\lambda})$. Clearly, for such ρ, there exists a unique $c(\rho)$ such that $\frac{V(c(\rho))}{V(c(\rho)+\frac{1}{2}) + \frac{\rho V(\frac{1}{2})}{1-\rho}} = 1 - \rho\lambda$. Therefore, $\frac{V(c(\rho_n))}{V(c(\rho_n)+\frac{1}{2}) + \frac{\rho V(\frac{1}{2})}{1-\rho}}$ converges to $\frac{V(c(\rho))}{V(c(\rho)+\frac{1}{2}) + \frac{\rho V(\frac{1}{2})}{1-\rho}}$ as ρ_n converges to ρ. Now, let's define function $m(.)$ by $m(c) = \frac{V(c)}{V(c+\frac{1}{2}) + \frac{\rho V(\frac{1}{2})}{1-\rho}}$. Then, $m(c(\rho_n))$ converges to $m(c(\rho))$.

Let's first assume that $\{\rho_n\}$ is an increasing sequence that converges to ρ. In this case, $\{c(\rho_n)\}$ is a decreasing and bounded sequence which therefore converges to some real number y^- such that $m(y^-) = m(c(\rho))$. This in turn implies that $y^- = c(\rho)$, since m is a strictly increasing function.

Now, let's suppose that $\{\rho_n\}$ is a sequence that decreases to ρ. Then, $\{c(\rho_n)\}$ is monotonic and converges to some real number y^+ such that $m(y^+) = m(c(\rho))$. Hence, m strictly increasing implies $y^- = c(\rho)$. Therefore, $y^- = y^+ = c(\rho)$ which concludes the proof that $c(.)$ is a continuous function.

Now, suppose $\rho = \frac{1}{\lambda}$. Then, $m(c(\rho_n))$ converges to the same limit as $1 - \rho_n\lambda$ which itself converges to 0. Hence, $m(c(\rho_n))$ converges to 0 which is in fact the value of $m(0)$. Therefore, when $\{\rho_n\}$ is an increasing sequence, $\{c(\rho_n)\}$ is a bounded and decreasing sequence that converges to some y such that $m(y) = m(0)$. Hence, by strict monotonicity of m, $\{c(\rho_n)\}$

converges to 0. But by definition of $c(.)$, $\lim_{x\uparrow\frac{1}{\lambda}} c(x) = 0$. Hence, for any sequence $\{\rho_n\}$ that increases to $\frac{1}{\lambda}$, $\{c(\rho_n)\}$ converges to $0 = \lim_{x\uparrow\frac{1}{\lambda}} c(x)$. This shows that $c(.)$ is left-continuous at $\frac{1}{\lambda}$.

Finally, suppose $\rho = 0$. It is left to show that for any sequence $\{\rho_n\}$ that decreases to ρ, $\{c(\rho_n)\}$ converges to $\lim_{x\downarrow 0} c(x)$. First, note that $\lim_{x\downarrow 0} c(x) = +\infty$. Note also that $\lim_{c\downarrow +\infty} m(c) = 1$. Now, suppose $\{\rho_n\}$ is a sequence that decreases to ρ. Then, $\{c(\rho_n)\}$ is a bounded and increasing sequence that converges to some z such that $\lim_{c\downarrow z} m(c) = 1 = \lim_{c\downarrow +\infty} m(c)$. Hence, $\{c(\rho_n)\}$ converges to $+\infty = \lim_{x\downarrow 0} c(x)$. Therefore, $c(.)$ is right-continuous at 0. This ends the proof that $c(.)$ is continuous on $(0, \frac{1}{\lambda})$. Finally, because $c(.)$ is monotonic, it follows that $c((0, \frac{1}{\lambda})) = (0 + \infty)$.

■

Theorem 18 *There exists $\rho^* \in (0, 1)$ such that if $\rho \leq \rho^*$, then in the best SPE, the economy stagnates whenever it becomes a dictatorship; if $\rho > \rho^*$, then in the best SPE the economy grows permanently whenever it becomes a dictatorship.*

Proof.

Let's fix λ. By Lemma 17, there exists $\tilde{\rho} \in (0, \frac{1}{\lambda})$ such that $c(\tilde{\rho}) = \frac{1}{2}$ and $c(\rho) \geq \frac{1}{2}, \forall \rho \leq \tilde{\rho}$. Clearly then, for any $\rho \leq \tilde{\rho}$, the solution to the best SPE problem of the dictatorship is $c^P = 0$ and $\Psi_D^c = \frac{V(\frac{1}{2})}{1-\rho}$. That is, any dictatorial economy with $\rho \leq \tilde{\rho}$ does not grow.

Note that $c(.)$ maps $[\tilde{\rho}, \frac{1}{\lambda})$ into $(0, \frac{1}{2})$. As in the proof of theorem 6, let's now define real functions g and f_D by $g(\rho) = \frac{1-\rho\lambda}{1-\rho}$ and $f_D(\rho) = \frac{U(1/2-c(\rho))}{U(1/2)}$. g is continuous and strictly decreasing in ρ. By lemmas 17 and 16, f_D is continuous and strictly increasing in ρ. Moreover, by the definition of $c(\tilde{\rho})$, $\frac{V(c(\tilde{\rho}))}{V(c(\tilde{\rho})+\frac{1}{2})+\frac{\tilde{\rho}V(\frac{1}{2})}{1-\tilde{\rho}}} = 1 - \tilde{\rho}\lambda$ and $c(\tilde{\rho}) = \frac{1}{2}$, so that $g(\tilde{\rho}) = \frac{1-\tilde{\rho}\lambda}{1-\tilde{\rho}} = \frac{1}{1-\tilde{\rho}} \frac{V(\frac{1}{2})}{V(1)+\frac{\tilde{\rho}V(\frac{1}{2})}{1-\tilde{\rho}}} > 0 = f_D(\tilde{\rho})$. Also, $g(\frac{1}{\lambda}) = 0 < 1 = f_D(\frac{1}{\lambda})$. ($c(\frac{1}{\lambda}) = 0$ implies $f(\frac{1}{\lambda}) = 1$). Therefore, there exists $\rho^* \in (\tilde{\rho}, 1/\lambda)$ s.t. $g(\rho^*) = f_D(\rho^*)$. For such ρ^*, $\frac{U(1/2-c(\rho^*))}{1-\rho^*\lambda} = \frac{U(1/2)}{1-\rho^*}$. For all $\rho \leq \rho^*$, $\frac{U(1/2-c(\rho^*))}{1-\rho^*\lambda} leq \frac{U(1/2)}{1-\rho^*}$ and $c_D^P = 0$, $\theta_D = 1$: the economy does not grow. For all $\rho > \rho^*$, $\frac{U(1/2-c(\rho^*))}{1-\rho^*\lambda} > \frac{U(1/2)}{1-\rho^*}$ and $c_D^P = c(\rho) \in (0, \frac{1}{2}]$, $\theta_D = 0$: the economy grows. Since it was argued earlier that the economy does not grow for $\rho < \tilde{\rho}$, this concludes the proof that a dictatorial economy grows iff $\rho > \rho^*$.

■

For given ρ and λ, let $\lambda^{S_t}\Psi_D^c$ and $\lambda^{S_t}\Psi_D^P$ denote payoffs to citizens and to the politician in power respectively in a dictatorial economy ending with state S_t. By an argument similar to that used to prove lemma 5 , the best SPE problem of an ξ- economy is stationary. It follows that payoffs to citizens in a best SPE starting with technological frontier S_t are given by $\lambda^{S_t}\Psi_{ND}^c$, where Ψ_{ND}^c is defined recursively as:

$$\Psi^c_{ND} = \max_{\{c^{HH}, c^P\}} U(c^{HH}) + \rho\lambda[(1-\xi)\Psi^c_{ND} + \xi\Psi^c_D]$$

s.t.

$$U(c^{HH}) + \rho\lambda((1-\xi)\Psi^c_{ND} + \xi\Psi^c_D) \geq \frac{U(1/2)}{1-\rho} \tag{35}$$

$$\Psi^P_{ND} = V(c^P) + \rho\lambda((1-\xi)\Psi^P_{ND} + \xi\Psi^P_D) \tag{36}$$

$$\Psi^P_{ND} \geq V(c^P + \frac{1}{2}) + \rho\xi\frac{V(1/2)}{1-\rho}. \tag{37}$$

The above operator maps the bounded interval $[\frac{U(1/2)}{1-\rho}, \frac{U(1/2)}{1-\rho\lambda}]$ into itself, satisfies Blackwell's conditions for a contraction mapping with modulus $\rho\lambda$ and is therefore a contraction mapping. This proves the existence of Ψ^c_{ND} as the fixed point of the defined contraction mapping.

Equivalently, $\Psi^c_{ND} = max\{\hat{\Psi}^c_{ND}, \frac{U(1/2)}{1-\rho}\}$, where $\hat{\Psi}^c_{ND}$ may be defined recursively as:

$$\hat{\Psi}^c_{ND} = \max_{\{c^{HH}, c^P\}} U(c^{HH}) + \rho\lambda[(1-\xi)\hat{\Psi}^c_{ND} + \xi\Psi^c_D]$$

s.t.

$$\frac{V(c^P) + \rho\lambda\xi\Psi^P_D}{1-\rho\lambda(1-\xi)} \geq V(c^P + \frac{1}{2}) + \rho\xi\frac{V(1/2)}{1-\rho}. \tag{38}$$

Because $\frac{V(c)+\rho\lambda\xi\Psi^P_D}{1-\rho\lambda(1-\xi)} \geq V(c+\frac{1}{2})$ is strictly increasing in c for given ρ, λ, ξ, any solution $\{c^{HH}_\xi, c^P_\xi\}$ to the sustainability constraint of the politician in an $\xi-$economy must hold with equality, unless $c^P_\xi = 0$.

Lemma 19 *Let λ be given. Let ρ^* be the threshold level of ρ that conditions growth in a dictatorial economy. Suppose $\rho > \rho^*$. Define c^P_D by $\Psi^P_D = \frac{V(c^P_D)}{1-\rho\lambda} = V(c^P_D + \frac{1}{2}) + \rho v^*_b$. For any $\xi \in [0,1]$, define c^P_ξ implicitly by:*

$$\begin{cases} \frac{V(c^P_\xi)+\rho\lambda\xi\Psi^P_D}{1-\rho\lambda(1-\xi)} = V(c^P_\xi + \frac{1}{2}) + \rho\xi v^*_b & if \ \frac{V(0)+\rho\lambda\xi\Psi^P_D}{1-\rho\lambda(1-\xi)} \leq V(0+\frac{1}{2}) + \rho\xi v^*_b \\ c^P_\xi = 0 \quad otherwise \ . \end{cases}$$

Then, $c^P_\xi < c^P_D$ for $\xi \in [0,1)$ and $c^P_1 = c^P_D$.

Proof.

Let's first show that $c^P_1 = c^P_D$. By definition, $V(c^P_1) + \rho\lambda\Psi^P_D = V(c^P_1 + \frac{1}{2}) + \rho v^*_b$, where $\Psi^P_D = \frac{V(c^P_D)}{1-\rho\lambda}$. Then, $V(c^P_1) + \rho\lambda\frac{V(c^P_D)}{1-\rho\lambda} - V(c^P_1 + \frac{1}{2}) = \rho v^*_b = \frac{V(c^P_D)}{1-\rho\lambda} - V(c^P_D + \frac{1}{2})$, which implies $V(c^P_1) - V(c^P_1 + \frac{1}{2}) = \frac{V(c^P_D)}{1-\rho\lambda}(1-\rho\lambda) - V(c^P_D + \frac{1}{2})$, or $V(c^P_1) - V(c^P_1+\frac{1}{2}) = V(c^P_D) - V(c^P_D + \frac{1}{2})$. But by strict concavity of V, $V(c) - V(c+\frac{1}{2})$ is strictly increasing in c. It follows that $c^P_1 = c^P_D > 0$. Now, for $\xi \in [0,1]$, define $z(\xi) = \frac{V(c^P_D)+\rho\lambda\xi\frac{V(c^P_D)}{1-\rho\lambda} - \rho\xi v^*_b(1-\rho\lambda+\rho\lambda\xi)}{1-\rho\lambda+\rho\lambda\xi}$. It is straightforward to show that $z^{'}(\xi) * [-\rho V_b^*(1-\rho\lambda+\rho\lambda\xi)] > 0$. That is, for all $\xi \in [0,1]$, $z^{'}(\xi) < 0$, given $\rho \geq \rho^* > 0$. But $z(1) = \frac{V(c_D^P)}{1-\rho\lambda} - \rho V_b^* = V(c_D^P + \frac{1}{2})$, where the last equality follows from the definition of c_D^P. Hence, $\forall \xi \in [0,1)$, $z(\xi) > V(c_D^P + \frac{1}{2})$, or $\frac{V(c_D^P)+\rho\lambda\xi\Psi_D^P}{1-\rho\lambda+\rho\lambda\xi} > V(c_D^P + \frac{1}{2}) + \rho\xi v_b^*$. If $\frac{V(0)+\rho\lambda\xi\Psi_D^P}{1-\rho\lambda(1-\xi)} \leq V(0+\frac{1}{2}) + \rho\xi v_b^*$, then it follows that there exists a unique $c_\xi^P \in [0, c_D^P)$ s.t $\frac{V(c_\xi^P)+\rho\lambda\xi\Psi_D^P}{1-\rho\lambda(1-\xi)} = V(c_\xi^P + \frac{1}{2}) + \rho\xi v_b^*$, since $\frac{V(y)+\rho\lambda\xi\Psi_D^P}{1-\rho\lambda+\rho\lambda\xi} - V(y+\frac{1}{2})$ is strictly increasing in y for ρ and ξ given. Now, suppose that $\frac{V(0)+\rho\lambda\xi\Psi_D^P}{1-\rho\lambda(1-\xi)} > V(0+\frac{1}{2}) + \rho\xi v_b^*$. Then, by definition, $c_\xi^P = 0$ and so, $c_\xi^P < c_D^P = c_1^P$. This ends the proof of the lemma. ■

Let $\tilde{\rho}^{\xi=0}$ be the value of ρ that conditions growth in a 0−economy, as defined in corollary 7. Note that $c_D^P > c_0^P$ implies that $\tilde{\rho}^{\xi=0} < \rho^*$.

Theorem 20 *Let $\lambda > 1$ be given. Suppose $\rho > \rho^*$, where ρ^* is the threshold value of ρ that conditions growth in a dictatorial economy. Then, the ξ−economy grows for any $\xi \in [0,1]$.*

Proof. $\xi \in [0,1]$ be arbitrary. Let $\rho > \rho^*$. Then, from the previous lemma, $c_D^P > c_\xi^P$. But from the proof of theorem 18, $c_D^P \in (0,1]$. Hence, to show that the ξ-economy grows, it suffices to show that $\frac{U(\frac{1}{2}-c_\xi^P)+\rho\lambda\xi\Psi_D^c}{1-\rho\lambda(1-\xi)} > \frac{U(\frac{1}{2})}{1-\rho}$. Now, let $c_j^{HH} = \frac{1}{2} - c_j^P, j \in \{D, \xi\}$. Then, $c_\xi^{HH} > c_D^{HH}$ and therefore it suffices to show that $\frac{U(c_D^P)+\rho\lambda\xi\Psi_D^c}{1-\rho\lambda(1-\xi)} > \frac{U(\frac{1}{2})}{1-\rho}$, which given $\Psi_D^c = \frac{U(c_D^P)}{1-\rho\lambda}$, holds iff $\frac{U(c_D^P)}{1-\rho\lambda} > \frac{U(\frac{1}{2})}{1-\rho}$. The last inequality is true by the proof of theorem 18, given $\rho > \rho^*$. Therefore, when $\rho > \rho^*$, the economy grows for any value of ξ. ■

The above theorem states that an economy that grows after becoming a dictatorship would also grow under any regime of ξ. It has also been shown that payoffs to the citizens continuously increase as the probability of falling into dictatorship decreases. These results constitute a caveat against using paragons of growing dictatorial economies to advocate dictatorship as an ideal political regime. Dictatorial economies that grow owe it to technocratic rulers. An economy endowed with impatient politicians would be worse off if it were to shift toward dictatorship.

The rest of this section examines the implications for growth under different regimes of ξ, when the discount factor is not sufficient high for growth to occur under dictatorship. Specifically, it is asked whether an economy known to stagnate under dictatorship (i.e. if $\rho \leq \rho^*$) would grow at all for any value of ξ. The answer is that in this case, an ξ-economy grows if and only if ξ is not too large. To obtain this last result, it is assumed that λ is not too large. Specifically, the rest of this section assumes that $\lambda \leq 2$ to ensure that if $\frac{V(x)+\rho\lambda\xi v_b^*}{1-\rho\lambda(1-\xi)} = V(x+\frac{1}{2}) + \rho\xi v_b^*$ for some x, then $\frac{V(x)+\rho\lambda\xi v_b^*}{1-\rho\lambda(1-\xi)} - \rho\xi v_b^*$ is decreasing in ξ. Recall that $\Psi_D^P = v_b^*$ when $\rho \leq \rho^*$. Hence when $1 < \lambda \leq 2$, as ξ increases, the sustainability constraint becomes tighter as politicians tend to ask a higher share of the consumption good in order to let growth occur. When λ is too large (greater than 2), it is no longer guaranteed that $\frac{V(x)}{1-\rho\lambda(1-\xi)} - \rho\xi v_b^* + \frac{\rho\lambda\xi v_b^*}{1-\rho\lambda(1-\xi)}$ is decreasing in ξ . In this case, it is not clear that optimal payments to politicians decrease with ξ, and so payoffs to citizens may increase or decrease as ξ increases. For the rest of this section, it is assumed that $1 < \lambda \leq 2$.

Lemma 21 *Suppose* $1 < \lambda \leq 2$*. Then,* $\forall \rho \in [0, \frac{1}{\lambda})$ *and* $\forall \xi \in [0,1]$, $\frac{(1-\rho+\rho\xi)(1-\rho\lambda+\rho\lambda\xi)-\rho\lambda\xi}{1-\rho\lambda+\rho\lambda\xi} > \frac{1-\rho\lambda-\frac{1}{\lambda}[1-\rho\lambda+\rho\lambda\xi][1-\rho\lambda+\rho\lambda\xi]}{1-\rho\lambda+\rho\lambda\xi}$.

Proof. It is easy to check that $\frac{(1-\rho+\rho\xi)(1-\rho\lambda+\rho\lambda\xi)-\rho\lambda\xi}{1-\rho\lambda+\rho\lambda\xi} > \frac{1-\rho\lambda-\frac{1}{\lambda}[1-\rho\lambda+\rho\lambda\xi][1-\rho\lambda+\rho\lambda\xi]}{1-\rho\lambda+\rho\lambda\xi}$ holds iff $2\rho(1-\xi) < \frac{1}{\lambda}$. Since $\rho(1-\xi) < 1$, $\lambda \leq 2$ guarantees $2\rho(1-\xi) < \frac{1}{\lambda}$, $\forall \rho \in [0, \frac{1}{\lambda})$ and $\forall \xi \in [0,1]$. ■

Lemma 22 *Let* $1 < \lambda \leq 2$*. Let* $\rho \in [0, \frac{1}{\lambda})$ *be such that* $\rho \leq \rho^*$*, where* ρ^* *is the value of* ρ *that conditions growth in a dictatorship. Let* $c_\xi^P \in [0, +\infty)$ *be defined implicitly by*

$$c_\xi^P = \begin{cases} 0 & \text{if } \frac{V(0)+\rho\lambda\xi v_b^*}{1-\rho\lambda(1-\xi)} > V(0+\frac{1}{2}) + \rho\xi v_b^* \\ \frac{V(c_\xi^P)+\rho\lambda\xi v_b^*}{1-\rho\lambda(1-\xi)} = V(c_\xi^P + \frac{1}{2}) + \rho\xi v_b^* & \text{otherwise.} \end{cases}$$

Then, for $\xi_2, \xi_1 \in [0,1]$ *such that* $\xi_2 > \xi_1$ *and* $c_{\xi_1}^P > 0$*, it holds that* $c_{\xi_2}^P > c_{\xi_1}^P > 0$*.*

Proof. Let $1 < \lambda \leq 2$. Fix $\rho \leq \rho^*$, so that payoffs to the politician in a dictatorship are given by $\Psi_D^P = v_b^*$. Fix $\xi \in [0,1]$. I first argue that c_ξ^P is well defined. Define function $l_\xi(.)$ by $l_\xi(c) = \frac{V(c)+\rho\lambda\xi v_b^*}{1-\rho\lambda(1-\xi)} - V(c+\frac{1}{2})$, for $c \in [0, +\infty]$. Clearly, $\lim_{x \to +\infty} l_\xi(x) = \lim_{x \to +\infty} V(x)[\frac{1}{1-\rho\lambda(1-\xi)} - 1] + \frac{\rho\lambda\xi V_b^*}{1-\rho\lambda(1-\xi)} = +\infty$. Therefore, if $\frac{V(0)+\rho\lambda\xi v_b^*}{1-\rho\lambda(1-\xi)} \leq V(0+\frac{1}{2}) + \rho\xi v_b^*$, then there exists $c \in [0, +\infty)$ s.t. $\frac{V(c)+\rho\lambda\xi v_b^*}{1-\rho\lambda(1-\xi)} = V(c+\frac{1}{2}) + \rho\xi v_b^*$. This proves that $c_\xi^P \in [0, +\infty)$ is well defined.

Now, consider $\xi_2 > \xi_1$ and such that $c_{\xi_1}^P > 0$. By definition, $c_{\xi_1}^P > 0$ implies that $\frac{V(c_{\xi_1}^P)+\rho\lambda\xi_1 v_b^*}{1-\rho\lambda(1-\xi_1)} = V(c_{\xi_1}^P + \frac{1}{2}) + \rho\xi_1 v_b^*$. Define function l by $l(\xi) = \frac{V(c_{\xi_1}^P)+\rho\lambda\xi v_b^*}{1-\rho\lambda(1-\xi)} - \rho\xi v_b^*$, for $\xi \in [0,1]$. It is straightforward to show that for $\xi \in [0,1]$, $l'(\xi)$ has the same sign as $q(\xi)$= $\frac{\lambda*[v_b^*(1-\rho\lambda)-V(c_{\xi_1}^P)]}{[1-\rho\lambda(1-\xi)]^2} - v_b^*$. But $\frac{V(c_{\xi_1}^P)}{1-\rho\lambda(1-\xi_1)} = V(c_{\xi_1}^P + \frac{1}{2}) + \rho\xi_1 v_b^* - \frac{\rho\lambda\xi_1 v_b^*}{1-\rho\lambda(1-\xi_1)} \geq V(\frac{1}{2}) + \rho\xi_1 v_b^* - \frac{\rho\lambda\xi_1 v_b^*}{1-\rho\lambda(1-\xi_1)} = v_b^*(1-\rho) + \rho\xi_1 v_b^* - \frac{\rho\lambda\xi_1 v_b^*}{1-\rho\lambda(1-\xi_1)} = \frac{v_b^*[(1-\rho+\rho\xi_1)(1-\rho\lambda+\rho\lambda\xi_1)-\rho\lambda\xi_1]}{1-\rho\lambda+\rho\lambda\xi_1}$. That is, $\frac{V(c_{\xi_1}^P)}{1-\rho\lambda(1-\xi_1)} \geq \frac{v_b^*[(1-\rho+\rho\xi_1)(1-\rho\lambda+\rho\lambda\xi_1)-\rho\lambda\xi_1]}{1-\rho\lambda+\rho\lambda\xi_1}$. But by lemma 3.1, $\lambda \leq 2$ implies $\frac{(1-\rho+\rho\xi_1)(1-\rho\lambda+\rho\lambda\xi_1)-\rho\lambda\xi_1}{1-\rho\lambda+\rho\lambda\xi_1} > \frac{1-\rho\lambda-\frac{1}{\lambda}[1-\rho\lambda+\rho\lambda\xi_1][1-\rho\lambda+\rho\lambda\xi_1]}{1-\rho\lambda+\rho\lambda\xi_1}$. Therefore, $\frac{V(c_{\xi_1}^P)}{1-\rho\lambda(1-\xi_1)} > v_b^* \frac{1-\rho\lambda-\frac{1}{\lambda}[1-\rho\lambda+\rho\lambda\xi_1]^2}{1-\rho\lambda+\rho\lambda\xi_1}$, which in turn implies that $q(\xi_1) < 0$. Hence, $l'(\xi_1) < 0$. Therefore, $\xi_2 > \xi_1$ implies $\frac{V(c_{\xi_1}^P)+\rho\lambda\xi_2 v_b^*}{1-\rho\lambda(1-\xi_2)} - \rho\xi_2 v_b^* < \frac{V(c_{\xi_1}^P)+\rho\lambda\xi_1 v_b^*}{1-\rho\lambda(1-\xi_1)} - \rho\xi_1 v_b^* = V(c_{\xi_1}^P + \frac{1}{2})$. That is, $\frac{V(c_{\xi_1}^P)+\rho\lambda\xi_2 v_b^*}{1-\rho\lambda(1-\xi_2)} < \rho\xi_2 v_b^* + V(c_{\xi_1}^P + \frac{1}{2})$. Hence, by strict monotonicity and continuity of l_{ξ_2} and by the fact that $\lim_{x \to +\infty} l_\xi(x) = +\infty$, there exists $c_{\xi_2}^P > c_{\xi_1}^P > 0$ such that $\frac{V(c_{\xi_2}^P)+\rho\lambda\xi_2 v_b^*}{1-\rho\lambda(1-\xi_2)} = \rho\xi_2 v_b^* + V(c_{\xi_2}^P + \frac{1}{2})$. This concludes the proof. ■

Corollary 23 *Let Let* $1 < \lambda \leq 2$*. Let* $\rho \in [0, \frac{1}{\lambda})$ *be such that* $\rho \leq \rho^*$*, where* ρ^* *is the value of* ρ *that conditions growth in a dictatorship. For all* $\xi \in [0,1]$*, let* c_ξ^P *be the solution to the best SPE problem of the* $\xi-$*economy. Then, for any* ξ_1 *and* ξ_2 *in [0,1] such that* $\xi_2 > \xi_1$*, it holds that* $c_{\xi_2}^P > c_{\xi_1}^P > 0$*.*

Proof. First note that in the best SPE problem of an ξ-economy, the sustainability constraint of the politician holds with equality if $c_\xi^P > 0$. To see why, suppose $c_\xi^P > 0$ and suppose that the sustainability constraint holds with strict inequality. Then, by decreasing the politician's consumption slightly, one improves citizens'payoffs without violating the politician's sustainability constraint. This contradicts that the initial allocation solves the best SPE problem. Now, because $0 = \frac{V(0)}{1-\rho\lambda} < V(\frac{1}{2})$, it follows that $c_0^P > 0$ for the 0-economy. Then from the lemma, it follows that for, any ξ_1 and ξ_2 in [0,1] such that $\xi_2 > \xi_1$, $c_{\xi_2}^P > c_{\xi_1}^P > 0$. Note that a direct implication of the corollary is that $\forall \xi \in [0,1], \exists$ unique $c_\xi^P \in (0,+\infty)$ s.t $\frac{V(c^P(\xi))+\rho\lambda\xi v_b^*}{1-\rho\lambda(1-\xi)} = V(c^P(\xi)+\frac{1}{2}) + \rho\xi v_b^*$. ■

Lemma 24 *Let $1 < \lambda \leq 2$. Let $\rho \in [0, \frac{1}{\lambda})$ be such that $\rho \leq \rho^*$, where ρ^* is the value of ρ that conditions growth in a dictatorship. For all $\xi \in [0,1]$, let $c^P(\xi)$ be defined by: $\frac{V(c^P(\xi))+\rho\lambda\xi v_b^*}{1-\rho\lambda(1-\xi)} = V(c^P(\xi)+\frac{1}{2}) + \rho\xi v_b^*$. Then, $c^P(.)$ is a continuous function of ξ.*

Proof. By the preceding corollary, $c^P(.)$ is a decreasing function of ξ. Hence, to show continuity, it suffices to show that for any monotonic sequence $\{\xi_n\}$ in $[0,1]$ that converges to some $\xi \in [0,1]$, $\{c^P(\xi_n)\}$ converges to $c^P(\xi)$. Let $\xi \in [0,1]$ be arbitrary. Let $\{\xi_n\}$ in $[0,1]$ be a monotonic sequence that converges to ξ. Then, $\{c^P(\xi_n)\}$ is a monotonic sequence that is bounded below by 0 and above by $c^P(1)$. Therefore, $\{c^P(\xi_n)\}$ converges to a unique y such that $\frac{V(y)+\rho\lambda\xi v_b^*}{1-\rho\lambda(1-\xi)} = V(y+\frac{1}{2}) + \rho\xi v_b^*$. Hence $\{c^P(\xi_n)\}$ converges to $c^P(\xi)$ and so $c^P(.)$ is a continuous function of ξ. ■

Theorem 25 *Let $1 < \lambda \leq 2$. Let $\rho \in [0, \frac{1}{\lambda})$ be such that $\rho \leq \rho^*$, where ρ^* is the value of ρ that conditions growth in a dictatorship. Let $\tilde{\rho}^{\xi=0}$ be the value of ρ that conditions growth in the $\xi = 0-$economy. Then, there exists $\xi^*(\rho) \in [0,1]$ such that:*

1. *If $\rho \leq \tilde{\rho}^{\xi=0}$, then the $\xi-$economy does not grow, $\forall \xi \in [0,1]$.*
2. *If $\rho > \tilde{\rho}^{\xi=0}$, then the the $\xi-$economy grows iff $\xi < \xi^*(\rho)$.*

Proof. Let $1 < \lambda \leq 2$. Let $\rho \in [0, \frac{1}{\lambda})$ be such that $\rho \leq \rho^*$. For any ξ in $[0,1]$, let $c^P(\xi)$ be defined by: $\frac{V(c^P(\xi))+\rho\lambda\xi v_b^*}{1-\rho\lambda(1-\xi)} = V(c^P(\xi)+\frac{1}{2}) + \rho\xi v_b^*$ and define $c^{HH}(\xi) = \frac{1}{2} - c^P(\xi)$. Recall the $\xi-$economy grows iff $\frac{U(c^{HH}(\xi))+\rho\lambda\xi u_b^*}{1-\rho\lambda(1-\xi)} > u_b^*$, which holds iff $\frac{U(c^{HH}(\xi))}{1-\rho\lambda} > \frac{U(\frac{1}{2})}{1-\rho}$. First suppose $\rho \leq \tilde{\rho}^{\xi=0}$. Then, the $0-$economy does not grow and $\frac{U(c^{HH}(0))}{1-\rho\lambda} \leq \frac{U(\frac{1}{2})}{1-\rho}$. But by lemma 22, $c^{HH}(\xi) < c^{HH}(0)$, $\forall \xi \in (0,1]$ and hence, $\frac{U(c^{HH}(\xi))}{1-\rho\lambda} < \frac{U(\frac{1}{2})}{1-\rho}$. That is, $\forall \xi \in [0,1]$, the economy never grows if $\rho \leq \tilde{\rho}^{\xi=0}$.

Now, suppose $\rho > \tilde{\rho}^{\xi=0}$, so that $\frac{U(c^{HH}(0))}{1-\rho\lambda} > \frac{U(\frac{1}{2})}{1-\rho}$. Recall from lemma 19 that for $\xi = 1$, $c^{HH}(1) = c_D^{HH}$, and $\frac{U(c_D^{HH})}{1-\rho\lambda} \leq \frac{U(\frac{1}{2})}{1-\rho}$, given $\rho \leq \rho^*$. Therefore, since $c^{HH}(.)$ is continuous and strictly increasing in ξ, there exists $\xi^*(\rho) \in (0,1]$ such that the economy grows iff $\xi < \xi^*(\rho)$. This ends the proof of the theorem.

■ To summarize, if $\rho \leq \tilde{\rho}^{\xi=0}$, then the ξ-economy does not grow for any value of ξ. If $\tilde{\rho}^{\xi=0} < \rho \leq \rho^*$, then the economy grows iff $\xi \leq \xi^*(\rho)$. If instead $\rho > \rho^*$, then the economy always grows. That is, a democratic economy which does not grow will also not grow if it were to shift toward dictatorship. A dictatorial economy which grows will also grow after democratization. For intermediate economies where agents are patient enough to allow growth under democracy but not sufficiently patient to allow growth under dictatorship, there exists a threshold probability of falling into dictatorship such that the economy grows if and only if it is below that threshold probability.

Conclusion

This paper has characterized necessary conditions for growth when technological progress is available and free, but requires the approval of self-interested politicians to be adopted. When they have a low discount factor, politicians in power find it optimal to stop technological progress in exchange for static rewards that the representative citizen does not control. This paper has shown that among economies where agents have the same degree of time preferences, those that grow are those with the strongest political institutions: the lowest probabilities of occurrence of a coup d'etat and the lowest probabilities of falling in the state of dictatorship. It has also been proved that citizens of a dictatorial economy will always gain from a one-shot transition to democracy, even in the case where the economy is already growing as a dictatorship. In fact, an economy that grows as a dictatorship will also grow for any given probability of falling into the state of dictatorship. Moreover, the smaller the probability of falling in the state of dictatorship, the smaller the minimal compensation that makes a politician in power willing to approve growth in a given period. This in turn implies that payoffs to the citizens increase as the likelihood of falling in the state of dictatorship decreases. Understanding the emergence of political structures is a next step that will further our understanding of growth mechanisms. For this purpose, extending the current framework by endogenizing the choice of political regimes appears a promising avenue for future research. In particular, it would be interesting to construct a model where agents' beliefs lead to two possible equilibria: one with a low probability of coups d'etat (or a low probability of transiting to dictatorship) and economic growth and one with a high probability of coups d'etat (or a high probability of transiting to dictatorship) and economic stagnation. I plan to undertake this task in the future.

Appendix

Proof. of Lemma 5

By lemma 3, best SPE sequences with no replacement of the initial politician are solutions to the SPE problem defined as:

$$\max_{\{c_\tau^{HH}, c_\tau^P, \theta_\tau\}_{\tau=0}^\infty \text{ feasible}} \sum_{\tau=0}^{\infty} \beta^\tau \lambda^{S_\tau} U[c_\tau^{HH}] \qquad s.t.$$

$$\sum_{\tau=0}^{\infty} \beta^\tau \lambda^{S_{t+\tau}} U[c_{t+\tau}^{HH}] \geq \frac{\lambda^{S_t}}{1-\beta} U(1/2), \forall t \geq 0$$

$$\sum_{\tau=0}^{\infty} \delta^\tau \lambda^{S_{t+\tau}} V[c_{t+\tau}^P + \frac{\theta_{t+\tau}}{2}] \geq \lambda^{S_t} V[c_t^P + \frac{1}{2}], \forall t \geq 0$$

Now, observe that if $\{c_\tau^{HH}, c_\tau^P, \theta_\tau\}_{\tau=0}^\infty$ is solution to the SPE problem, then for all $s \geq 0$, $\{c_\tau^{HH}, c_\tau^P, \theta_\tau\}_{\tau=s}^\infty$ must solve the date s SPE problem defined by:

$$\max_{\{c_{s+\tau}^{HH}, c_{s+\tau}^P, \theta_{s+\tau}\}_{\tau=0}^\infty \text{ feasible}} \sum_{\tau=0}^{\infty} \beta^\tau \lambda^{S_{s+\tau}} U[c_{s+\tau}^{HH}]$$

$$\sum_{\tau=0}^{\infty} \beta^\tau \lambda^{S_{s+t+\tau}} U[c_{s+t+\tau}^{HH}] \geq \frac{\lambda^{S_{s+t}}}{1-\beta} U(1/2), \forall t \geq 0$$

$$\sum_{\tau=0}^{\infty} \delta^\tau \lambda^{S_{s+t+\tau}} V[c_{s+t+\tau}^P + \frac{\theta_{s+t+\tau}}{2}] \geq \lambda^{S_{s+t}} V[c_{s+t}^P + \frac{1}{2}], \forall t \geq 0$$

$$S_s \text{ given}, S_{s+\tau+1} = S_{s+\tau} + 1 - \theta_{s+\tau}, \forall \tau \geq 0.$$

After using the law of motion of the technological frontier and simplifying S_{s+t} from both sides of the inequality constraints, the above problem reduces to:

$$\max_{\{c_{s+\tau}^{HH}, c_{s+\tau}^P, a_{s+\tau}, \theta_{s+\tau}\}_{\tau=0}^\infty} \lambda^{S_s} [U[c_s^{HH}] + \sum_{\tau=1}^{\infty} \beta^\tau \lambda^{a_s + \ldots + a_{s+\tau-1}} U[c_{s+\tau}^{HH}]]$$

s.t.

$$\sum_{\tau=0}^{\infty} \beta^{\tau} \lambda^{a_{s+t}+\ldots+a_{s+t+\tau-1}} U[c^{HH}_{s+t+\tau}] \geq \frac{U(1/2)}{1-\beta}, \forall t \geq 0 \tag{41}$$

$$\sum_{\tau=0}^{\infty} \delta^{\tau} \lambda^{a_{s+t}+\ldots+a_{s+t+\tau-1}} V[c^{P}_{s+t+\tau} + \frac{1}{2} - \frac{a_{s+t+\tau}}{2}] \geq V[c^{P}_{s+t} + \frac{1}{2}], \forall t \geq 0, \tag{42}$$

$$\theta_{s+\tau} = 1 - a_{s+\tau}, \forall \tau \geq 0, \tag{43}$$

S_s given, $S_{s+\tau+1} = S_{s+\tau} + 1 - \theta_{s+\tau}$, for all $\tau \geq 0$ and $\{c^{HH}_{s+t+\tau}, c^{P}_{s+t+\tau}, \theta_{s+t+\tau}\}_{\tau=0}^{\infty}$ feasible.

■

Now, for a contradiction, suppose some date $s+1$ best SPE sequence $\{\hat{c}^{HH}_{s+1+\tau}, \hat{c}^{P}_{s+1+\tau}, \hat{\theta}_{s+1+\tau}\}_{\tau=0}^{\infty}$ yields strictly higher payoff to citizens than some date s best SPE sequence $\{c^{HH}_{s+\tau}, c^{P}_{s+\tau}, \theta_{s+\tau}\}_{\tau=0}^{\infty}$. Then, the new sequence $\{c^{*HH}_{s+\tau}, c^{*P}_{s+\tau}, \theta^{*}_{s+\tau}\}_{\tau=0}^{\infty} \equiv \{\hat{c}^{HH}_{s+1+\tau}, \hat{c}^{P}_{s+1+\tau}, \hat{\theta}_{s+1+\tau}\}_{\tau=0}^{\infty}$ satisfies the inequality constraints of the date s best SPE problem and yields strictly higher date s payoff to citizens, which contradicts that the original date s sequence was a best SPE sequence.

By a symmetric argument, no date s best SPE sequence yields a strictly higher payoff than a date $s+1$ best SPE sequence. Therefore, best SPE sequences in all periods yield the same payoff to citizens.

Now, define $A_{\tau} = \{c^{HH}_{\tau}, c^{P}_{\tau}, \theta_{\tau}\}$, for all $\tau \geq 0$. It therefore follows that there exists a best SPE sequence in which $\{A_{\tau}\}_{\tau=0}^{\infty} = \{A_{\tau}\}_{\tau=1}^{\infty} = \{A_{\tau}\}_{\tau=2}^{\infty} = \ldots = \{A_{\tau}\}_{\tau=s}^{\infty}$ ... implying $\{A_{\tau}\} = A^{*}$, $\forall \tau \geq 0$. That is, there exists a best SPE sequence that is stationary: $\{c^{HH}_{\tau}, c^{P}_{\tau}, \theta_{\tau}\} = \{c^{*HH}_{\tau}, c^{*P}_{\tau}, \theta^{*}_{\tau}\}$, $\forall \tau \geq 0$.

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